

On cosmic hair and “de Sitter breaking” in linearized quantum gravity

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Abstract

We quantize linearized Einstein-Hilbert gravity on de Sitter backgrounds in a covariant gauge. We verify the existence of a maximally-symmetric (i.e. de Sitter-invariant) Hadamard state Ω for all globally hyperbolic de Sitter backgrounds in all spacetime dimensions $D \geq 4$ by constructing the state’s 2-point function in closed form. This 2-pt function is explicitly maximally symmetric. We prove an analogue of the Reeh-Schlieder theorem for linearized gravity. Using these results we prove a cosmic no-hair theorem for linearized gravitons: for any state in the Hilbert space constructed from Ω , the late-time behavior of local observable correlation functions reduces to those of Ω at an exponential rate with respect to proper time. We also provide the explicitly maximally-symmetric graviton 2-pt functions in a class of generalized de Donder gauges which are suitable for use in non-linear perturbation theory. Along the way we clarify a few technical aspects which led previous authors to conclude that these 2-pt functions do not exist.

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1 Introduction and summary

Perturbative quantum gravity on de Sitter backgrounds is of considerable interest. de Sitter spacetime is the maximally-symmetric model of an expanding cosmology and may be used to approximate our universe both in the inflationary era as well as our current epoch of accelerated expansion. At the level of phenomenology, perturbative gravitons on de Sitter may be used to study tensor fluctuations in the cosmic microwave background (CMB) [1]. Of more theoretical interest, previous studies have led many authors to conjecture interesting infrared (IR) effects associated to de Sitter gravitons. It has been argued that even in perturbation theory one may find large quantum (i.e., loop) IR corrections [2, 3, 4, 5, 6], the dynamical screening of the effective cosmological constant [7, 8, 9, 10], and other destabilizing phenomena [11, 12, 13]. An interesting feature of these effects is that they appear to break de Sitter symmetry. This has caused some controversy as the precise mechanism by which de Sitter symmetry is broken has not been adequately explained. To this day both the existence and role of a maximally-symmetric (a.k.a. de Sitter-invariant) graviton state is heatedly debated.

It is a general expectation of quantum field theory (QFT) in curved spacetime [14] that when the background spacetime admits isometries the associated isometry group should play

a crucial role in organizing the theory, much like how the Poincaré group organizes QFT in Minkowski spacetime. This expectation is realized for QFTs with a mass gap on a de Sitter background. For instance, it is known that interacting massive scalar QFTs enjoy a unique maximally-symmetric state [15, 16, 17, 18]. This state may be interpreted as the interacting “Bunch-Davies vacuum” in the cosmological chart or as the thermal state at the de Sitter temperature in the static chart. Most important is the role this maximally-symmetric state plays in the “cosmic no-hair theorem” of [15, 16] which states that the expectation values of local observables in a *generic* state limit to those of the maximally-symmetric state in the asymptotic regions of de Sitter. This makes the maximally-symmetric state the most relevant state when studying the asymptotic behavior of expectation values of the theory.

It is of great interest to determine under what circumstances a similar cosmic no-hair theorem exists for perturbative quantum gravity. In classical gravity, asymptotically de Sitter spaces are known to be non-linearly stable under an appropriate class of perturbations [19, 20]. Moreover, a no-hair theorem of Wald [21] states that initially expanding homogeneous solutions to vacuum Einstein’s equations with positive cosmological constant exponentially evolve toward locally de Sitter spaces. These classical results suggest that at least at tree-level quantum gravity should admit a cosmic no-hair theorem very similar to that of the dS QFTs described above. In particular, these results imply the existence of a maximally-symmetric graviton state which acts as the attractor state for local observables at late times. This is further supported by the recent non-perturbative semi-classical analysis of [22].

The goal of this paper is to make precise the cosmic no-hair theorem for *linearized* quantum gravity on de Sitter. We quantize linearized Einstein-Hilbert gravity in $D \geq 4$ spacetime dimensions on a de Sitter background. We employ an algebraic approach to quantization which allows us to quantize the theory in covariant gauges, greatly facilitate our computations, and make our results transparent. The algebraic approach also allows us to simultaneously discuss the quantization on various de Sitter charts which may be taken to define globally hyperbolic spacetimes in their own right. Excepting the Appendix all computations are performed in Lorentz signature, though in reality most of our manipulations are insensitive to the metric signature. We have four main results which we summarize now:

- R1. We verify the existence of a maximally-symmetric state Ω by constructing the graviton 2-pt correlation function of the state $\langle h_{\mu\nu}(x)h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_{\Omega}$ explicitly in transverse traceless gauge. This 2-pt function may be written in closed form in terms of maximally-symmetric bi-tensors and is manifestly maximally symmetric. We verify that the state Ω is Hadamard and satisfies the positivity (a.k.a. “unitarity”) condition.
- R2. We prove an analogue of the Reeh-Schlieder theorem for linearized gravity (Theorem 4.2). The Reeh-Schlieder theorem states that the set of states generated from Ω by the algebra of local observables of a contractible region is dense on the Hilbert space. The Reeh-Schlieder property is a remarkable attribute of *local* quantum field theories and in the context of gravity is clearly special to the linear theory. Nevertheless, this theorem allows us to prove the next result with some rigour.
- R3. We prove a cosmic no-hair theorem for linearized gravity (Theorem 4.6) which states

the following. Let Ψ be a state on the Hilbert space constructed from Ω via the GNS construction and let A be an observable whose support is compact and contractible. At sufficiently late times the expectation value $\langle A(\tau) \rangle_\Psi$ approaches $\langle A(\tau) \rangle_\Omega$ rapidly:

$$|\langle A(\tau) \rangle_\Psi - \langle A(\tau) \rangle_\Omega| < c e^{-2\tau}. \quad (1.1)$$

Here c is a finite constant and τ is the proper time separation from any reference point in the past. This result follows rather simply from the fact that the expectation values of *local observables* in the state Ω obey cluster decomposition at large timelike and achronal separations.

This no-hair theorem may be regarded as a quantum formulation, for linearized perturbations, of the classical stability theorems of Freidrich [19] and Anderson [20]. We emphasize that this result holds in linearized quantum gravity, i.e. we do not consider self-interactions or coupling to matter fields. As such the no-hair theorem does not directly constrain the remarkable effects mentioned in the first paragraph, though we believe it provides a valuable perspective. Our results *do* show that “de Sitter breaking” [23, 24] is not a phenomenon of linearized quantum gravity.

Our last result is an aside and is not used to obtain results (R2) or (R3):

R4. We compute the 2-pt function of Ω in the one-parameter class of generalized de Donder gauges which satisfy the gauge condition

$$\nabla^\nu h_{\mu\nu}(x) - \frac{\beta}{2} \nabla_\mu h^\nu{}_\nu(x) = 0, \quad \beta \in \mathbb{R}. \quad (1.2)$$

This class of gauges is of interest because the gauge condition is generally covariant and may be imposed in non-linear perturbation theory (unlike transverse traceless gauge). We obtain a manifestly maximally-symmetric expression for the 2-pt function for all but a discrete set of values for the gauge parameter β . Our result agrees with the expression that may be obtained in $D = 4$ from a limit of the “covariant gauge” 2-pt function of [25, 26, 27].

There is a large literature debating the existence of manifestly maximally-symmetric graviton 2-pt functions – see e.g. [28, 29, 23, 24, 30, 31, 27]. We do not wish to sink into the depths of this *melée* – our results (R1) and (R4) speak for themselves – but we should note two points of contact with this literature. First, although our 2-pt functions are computed in Lorentz signature they agree with the analytic continuation of Euclidean 2-pt functions constructed on the Euclidean sphere S^D . Several previous authors have utilized this technique [32, 13, 26, 25, 33], but its validity has been debated [34, 24]. Since the health of our results has been verified in Lorentz signature, we conclude that at least for the cases we consider no pathologies arise from this analytic continuation process.

Second, we note that our results (R1) and (R4) are technically in conflict with the claims of [35] and [36]: these works claim that there do not exist maximally-symmetric solutions to the graviton 2-pt Schwinger-Dyson equations in transverse traceless or generalized de Donder gauges. This conflict stems from the fact that these gauge conditions do not completely fix

the gauge freedom. Refs. [35, 36] did not explore the full space of solutions consistent with these partial gauge conditions. We discuss this in detail in §3.5. This conflict aside, we emphasize that the *state* defined by the less-symmetric 2-pt functions of [35, 36] is equivalent to the maximally-symmetric state Ω as probed by all local observables. For certain classes of observables this fact has been pointed out before [30, 31, 27, 37].

The remainder of this paper is organized as follows. We begin in §2 by introducing preliminary material including the basic attributes of de Sitter space and linearized gravity as well as our quantization scheme. We compute the 2-pt function of Ω in transverse traceless gauge in §3. The two theorems listed above are presented in §4. Then in §5 we compute the 2-pt function of Ω in generalized de Donder gauge.

2 Preliminaries

2.1 de Sitter space

In this paper we consider Einstein-Hilbert gravity in D spacetime dimensions with positive cosmological constant $\Lambda > 0$. The classical theory may be defined by its action

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^D x \sqrt{-g(x)} (R(g) - 2\Lambda), \quad (2.1)$$

or equivalently by its equations of motion

$$G_{\mu\nu}(g) + \Lambda g_{\mu\nu}(x) = 0, \quad (2.2)$$

where $R(g)$ and $G_{\mu\nu}(g)$ are the Ricci scalar and Einstein tensor constructed from the metric $g_{\mu\nu}(x)$ respectively. de Sitter space is the maximally symmetric solution to these equations. The D -dimensional de Sitter manifold dS_D may be defined as the single-sheet hyperboloid in an $\mathbb{R}^{D,1}$ embedding space:

$$dS_D := \{X \in \mathbb{R}^{D,1} \mid X \cdot X = \ell^2\}. \quad (2.3)$$

The de Sitter radius ℓ is related to the cosmological constant via

$$\Lambda = \frac{(D-1)(D-2)}{2\ell^2}. \quad (2.4)$$

The full de Sitter manifold has the topology $\mathbb{R} \times S^{D-1}$ where \mathbb{R} is the timelike direction; it has two conformal boundaries \mathcal{I}^\pm which are Euclidean spheres S^{D-1} . From the embedding space description it is manifest that the isometry group of de Sitter, a.k.a. the de Sitter group, is $SO(D, 1)$.

de Sitter space may be described by a number of coordinate charts, the most common of which include the global, Poincaré (a.k.a. cosmological), and static charts. Although these charts cover different portions of the full de Sitter manifold each chart defines a globally

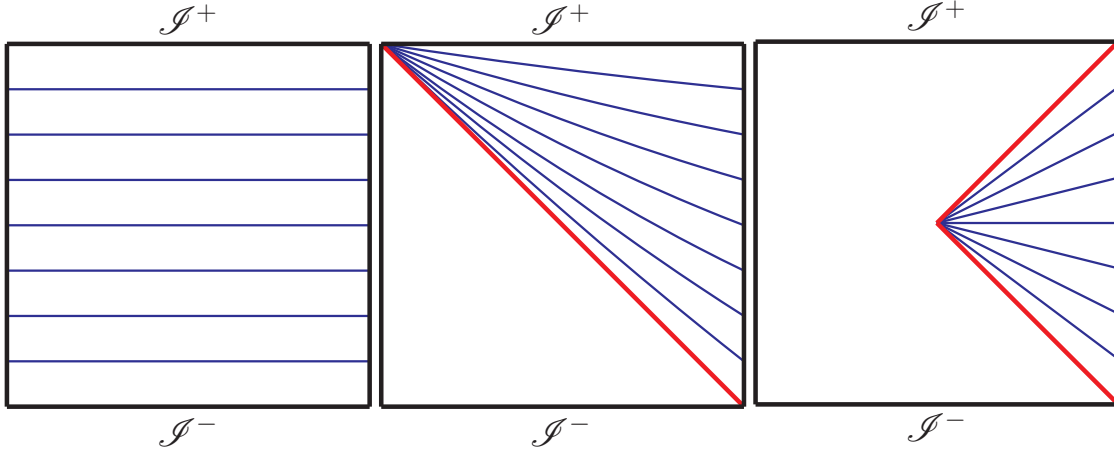


Figure 1: De Sitter coordinate charts depicted on the Carter-Penrose diagram. **Left:** The global de Sitter chart with constant t hypersurfaces depicted by blue lines. **Center:** The Poincaré chart with constant η hypersurfaces depicted by blue lines. This chart covers only the region above the cosmological horizon (solid red line). **Right:** The static chart with constant u hypersurfaces depicted by blue lines. This chart covers only the region enclosed by past and future cosmological horizons (solid red lines).

hyperbolic spacetime in it's own right. Each chart is depicted on the de Sitter Carter-Penrose diagram in Fig. 1. The global chart covers the entire manifold and is given by the line element

$$\frac{ds^2}{\ell^2} = -dt^2 + (\cosh t)^2 d\Omega_{D-1}^2, \quad t \in \mathbb{R}. \quad (2.5)$$

Here $d\Omega_{D-1}^2$ is the line element on a unit S^{D-1} . In this chart equal time hypersurfaces are spheres of radius $\ell^2(\cosh t)^2$. The Poincaré chart is given by

$$\frac{ds^2}{\ell^2} = \frac{1}{\eta^2} [-d\eta^2 + d\vec{x} \cdot d\vec{x}], \quad \eta \in (-\infty, 0), \quad (2.6)$$

where \vec{x} denote vectors in \mathbb{R}^{D-1} . This chart covers half of the full de Sitter manifold and describes a cosmology with open spatial slices which expand exponentially with increasing proper time. Finally, the static chart of de Sitter may be described by the line element

$$\frac{ds^2}{\ell^2} = -\cos^2 \theta du^2 + d\theta^2 + \sin^2 \theta d\Omega_{D-2}^2, \quad u \in \mathbb{R}, \quad \theta \in \left[0, \frac{\pi}{2}\right). \quad (2.7)$$

The static chart covers an even smaller region of de Sitter: it is the largest region in which a de Sitter “boost” Killing vector field ∂_u is timelike and future-directed. Further discussion of de Sitter charts as well as other basic features of de Sitter may be found in [38, 39, 40].

In order to conduct physics on de Sitter we need measures of distance and the causal structure. As usual an invariant notion of distance between two points $x_1, x_2 \in dS_D$ is provided by the signed, squared geodesic distance $\text{geod}(x_1, x_2)$, though it is more convenient

to package this information in the $SO(D, 1)$ -invariant “embedding distance” [41]:

$$Z(x_1, x_2) := \frac{X_1 \cdot X_2}{\ell^2} = \begin{cases} \cos \left[\ell^{-1} \sqrt{|\text{geod}(x_1, x_2)|} \right] & \text{for spacelike points} \\ \cosh \left[\ell^{-1} \sqrt{|\text{geod}(x_1, x_2)|} \right] & \text{for timelike and achronal points} \end{cases} \quad (2.8)$$

The embedding distance satisfies

- i) $Z(x_1, x_2) \in [-1, 1)$ for spacelike separation,
- ii) $Z(x_1, x_2) = 1$ for null separation,
- iii) $Z(x_1, x_2) > 1$ for timelike separation, and
- iv) $Z(x_1, x_2) < -1$ for achronal separation.

The embedding distance is shown for various configurations in Fig. 2. In order to describe the causal structure it is useful to introduce the function $s(x_1, x_2)$ [41]:

$$s(x_1, x_2) := \begin{cases} +1 & \text{if } x_1 \in J^+(x_2) \\ -1 & \text{if } x_1 \in J^-(x_2) \\ 0 & \text{else} \end{cases}, \quad (2.9)$$

where $J^{+(-)}(x)$ denotes the causal future (past) of x . Therefore the quantity $Z(x_1, x_2) + i\epsilon s(x_1, x_2)$, which is invariant under $SO_0(D, 1)$ rather than the full isometry group $SO(D, 1)$, provides both the geodesic distance between points as well as their causal relationship. The following expressions give the embedding distance in the coordinate charts described above:

$$Z(x_1, x_2) = \begin{cases} -\sinh t_1 \sinh t_2 + \cosh t_1 \cosh t_2 \cos \Omega_{12} & \text{(global)} \\ 1 - \frac{|\vec{x}_1 - \vec{x}_2|^2 - (\eta_1 - \eta_2)^2}{2\eta_1 \eta_2} & \text{(Poincaré)} \\ \cos \theta_1 \cos \theta_2 \cosh(u_1 - u_2) + \sin \theta_1 \sin \theta_2 \cos \omega_{12} & \text{(static)} \end{cases} \quad (2.10)$$

2.2 Linearized gravity

To obtain the theory of linear metric perturbations on de Sitter space we let

$$g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x) + \sqrt{8\pi G} h_{\mu\nu}(x), \quad (2.11)$$

where $g_{\mu\nu}(x)$ is a de Sitter metric and $h_{\mu\nu}(x)$ a symmetric perturbation. All tensor indices are raised/lowered with the background metric $g_{\mu\nu}(x)$. The linearized equations of motion are¹

$$\begin{aligned} L_{\mu\nu}^{(1)}(h) &:= G_{\mu\nu}^{(1)}(h) + \Lambda h_{\mu\nu}(x) \\ &= \frac{1}{2} \square h_{\mu\nu}(x) + \frac{1}{2} \nabla_\mu \nabla_\nu h(x) - \nabla_{(\mu} \nabla^\lambda h_{\nu)\lambda}(x) - \frac{1}{\ell^2} h_{\mu\nu}(x) \\ &\quad + \frac{1}{2} \left(\nabla^\alpha \nabla^\beta h_{\alpha\beta}(x) - \square h(x) + \frac{3-D}{\ell^2} h(x) \right) g_{\mu\nu}(x) \\ &= 0, \end{aligned} \quad (2.12)$$

¹ The authors of [37, 42] refer to $L_{\mu\nu}^{(1)}(h)$ defined in (2.12) as the linearized Einstein tensor.

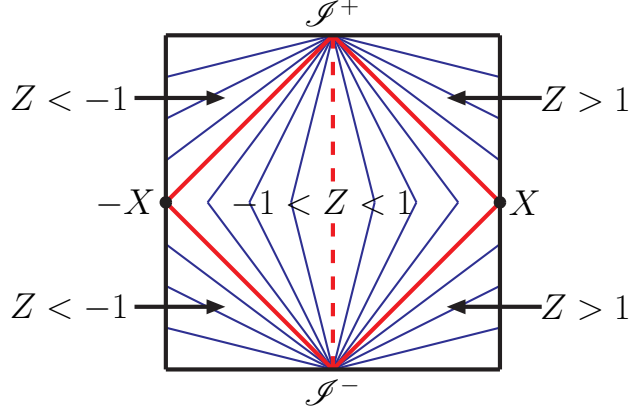


Figure 2: The embedding distance $Z(x, y)$ on de Sitter. The point x (equivalently X) and it's antipodal point $-X$ are shown by dots. Thin blue lines depict constant $Z(x, y)$ contours, thick red lines denote the contours $Z(x, y) = \pm 1$, and the dashed red line denotes $Z(x, y) = 0$.

where $h(x) = h^\nu{}_\nu(x)$.² The operator $L_{\mu\nu}^{(1)}(h)$ is symmetric, transverse $\nabla^\mu L_{\mu\nu}^{(1)}(h) = 0$, and linear in the usual sense: $L_{\mu\nu}^{(1)}(\alpha h_{\mu\nu} + \beta \gamma_{\mu\nu}) = \alpha L_{\mu\nu}^{(1)}(h) + \beta L_{\mu\nu}^{(1)}(\gamma)$ for $\alpha, \beta \in \mathbb{C}$. It is also self-adjoint in the sense that for tensors $f_{\alpha\beta}(x)$, $q_{\mu\nu}(x)$ such that $f_{\alpha\beta}(x)q_{\mu\nu}(x)$ has compact support then

$$\int d^D \sqrt{-g} q^{\mu\nu}(x) L_{\mu\nu}^{(1)}(f) = \int d^D \sqrt{-g} f^{\mu\nu}(x) L_{\mu\nu}^{(1)}(q). \quad (2.13)$$

Linearized gravity has the gauge symmetry

$$h_{\mu\nu}(x) \rightarrow h_{\mu\nu}(x) + \mathcal{L}_\xi g_{\mu\nu}(x) = h_{\mu\nu}(x) + 2\nabla_{(\mu} \xi_{\nu)}(x). \quad (2.14)$$

As a result the linearized metric perturbations are defined only up to the equivalence class whose orbit is given by (2.14). The equations of motion are gauge-invariant in the sense that $L_{\mu\nu}^{(1)}(\mathcal{L}_\xi g) = 0$. As for curvature quantities, it follows that the linearized Einstein tensor $G_{\mu\nu}^{(1)}(h)$ is not gauge-invariant. The linearized Weyl tensor $C^{(1)\alpha}{}_{\beta\gamma\delta}(h)$ is gauge invariant

$$C^{(1)\alpha}{}_{\beta\gamma\delta}(\mathcal{L}_\xi g) = 0, \quad (2.15)$$

and is also invariant under linearized Weyl transformations

$$C^{(1)\alpha}{}_{\beta\gamma\delta}(\omega g_{\mu\nu}) = 0, \quad (2.16)$$

for arbitrary $\omega(x)$. Because the Weyl tensor of the de Sitter background vanishes these equations are valid for any index configuration. An explicit formula for $C^{(1)\alpha\beta}{}_{\gamma\delta}(h)$ is

$$C^{(1)\alpha\beta}{}_{\gamma\delta}(h) := \sqrt{8\pi G} \Upsilon_{[\gamma\delta][\mu\nu]}^{[\alpha\beta][\rho\sigma]} (\nabla_\rho \nabla^\mu + \ell^{-2} \delta_\rho^\mu) h^\nu{}_\sigma, \quad (2.17)$$

$$\Upsilon_{\gamma\delta\mu\nu}^{\alpha\beta\rho\sigma} := -2 \left(\delta_\mu^\alpha \delta_\nu^\beta \delta_\gamma^\rho \delta_\delta^\sigma + \frac{4}{(D-2)} \delta_\mu^\alpha \delta_\gamma^\beta \delta_\delta^\rho \delta_\nu^\sigma + \frac{2}{(D-2)(D-1)} \delta_\gamma^\alpha \delta_\delta^\beta \delta_\mu^\rho \delta_\nu^\sigma \right). \quad (2.18)$$

²When there is no risk of confusion we will omit tensor indices on arguments. This does not mean the argument is simply the trace of the tensor.

Many useful lemmas, theorems, are formulae for linearized gravity in vacuum cosmological spacetimes are presented in [42].

2.3 Quantization

In this section we outline our quantization procedure which is an example of the algebraic approach to QFT in curved spacetime [43].³ Although there are several advantages to this approach (see e.g. the discussions in [45, 46, 14]), the immediate advantage for our purposes is that it allows us to quantize linearized gravity in covariant gauges which in turn allow us to preserve manifest de Sitter symmetry throughout our computations. These gauges are not a complete gauge fixing and are analogous to Lorentz gauge in vector gauge theories. The algebraic quantization of linearized gravity on cosmological backgrounds has been treated previously by Fewster and Hunt [42].

A second advantage of the algebraic approach is that it allows us to discuss in a unified manner the quantization of gravity in each of the de Sitter backgrounds described in §2.1. Other tactics such as path integral or “mode quantization” approaches, if performed in any detail, would require us to work separately on each background as each admits different sets of solutions to the classical equations of motion. Various technical issues then arise in comparing the resulting Hilbert spaces. In any case, we will refer the background de Sitter spacetime simply as “ dS_D ” which implicitly includes a choice of chart and will make chart-specific comments as needed. We also adopt the notation

$$\int_x F(x) := \int d^D x \sqrt{-g(x)} F(x), \quad (2.19)$$

where similarly the volume element is that of the background of interest. The integrands of such expressions will always have compact support.

Observables

Local observables of the quantum theory are the analogues of local, gauge-invariant quantities of the classical theory. Observables constructed directly from the metric perturbation are of the form [42]

$$h(f) := \int_x f^{\mu\nu}(x) h_{\mu\nu}(x), \quad f \in \mathcal{T}_T, \quad (2.20)$$

where $f^{\mu\nu}(x)$ is a “test function” that is compactly supported, symmetric, and transverse, i.e., it belongs to the class

$$\mathcal{T}_T := \{f^{\mu\nu} \in C_0^\infty(T_0^2(dS_D)) \mid f^{\mu\nu} = f^{\nu\mu}, \nabla_\mu f^{\mu\nu} = 0\}. \quad (2.21)$$

In principal compact support can include support on an entire S^{D-1} Cauchy surface in global dS, so long as the support in the timelike direction is compact. The compactness and transversality of $f^{\mu\nu}(x)$ guarantee that $h(f)$ is gauge-invariant. In addition these observables satisfy

³A thorough treatment of the algebraic approach to QFT in Minkowski space is presented in [44].

O1. Linearity: $h(\alpha f^{\mu\nu} + \beta p^{\mu\nu}) = \alpha h(f) + \beta h(p), \quad \alpha, \beta \in \mathbb{C},$

O2. Hermiticity: $h^\dagger(f) = h(f^*),$

O3. Equations of motion: $h(L^{(1)}(f)) = 0,$

O4. Locality (canonical commutation relations): $[h(f), h(p)] = i\Delta(f, p),$

where $\Delta(f, p)$ is the smeared commutator function, i.e. the unique advanced-minus-retarded solution to the equations of motion.

One may also consider observables constructed from the linearized Weyl tensor:

$$C^{(1)}(v) := \int_x v^{\mu\nu\rho\sigma}(x) C_{\mu\nu\rho\sigma}^{(1)}(h), \quad v \in C_0^\infty(T_0^4(dS_D)). \quad (2.22)$$

Smeared “Wick powers” of the linearized Weyl tensor such as $C_{\alpha\beta\gamma\delta}^{(1)} C^{(1)\mu\nu\rho\sigma}(x)$, $C_{\alpha\beta\gamma\delta}^{(1)} C^{(1)\gamma\delta\mu\nu}(x)$, etc., are also local observables provided an ordering prescription for defining composite operators (such as that of [47] or [48]).

Consider the set of observables of the forms (2.20), (2.22), as well as their associated gauge-invariant Wick powers, all constructed from test functions whose support is contained in the compact region $\mathcal{O} \subset dS_D$. Along with the identity element, the polynomial algebra generated by finite sums of finite products of members of this set form a unital $*$ -algebra $\mathcal{A}(\mathcal{O})$; the union of the algebras of all such open sets on dS_D to define the total algebra of local observables

$$\mathcal{A}(dS_D) = \cup_{\mathcal{O}} \mathcal{A}(\mathcal{O}). \quad (2.23)$$

States

A quantum state Ψ is a linear functional on the algebra of observables $\Psi : \mathcal{A}(dS_D) \rightarrow \mathbb{C}$ which additionally satisfies

S1. Normalization: $\langle 1 \rangle_\Psi = 1,$

S2. Positivity: (a.k.a. “unitarity”): $0 \leq \langle A^\dagger A \rangle_\Psi < \infty, \quad \forall A \in \mathcal{A}(dS_D),$

S3. Ψ must be of the Hadamard type.

The first two requirements are familiar and are necessary for Ψ to admit a quantum mechanical interpretation as providing conditional probabilities. The third criteria is in some respects a choice; it is a regularity condition which assures that the states we consider i) reproduce familiar Minkowski space physics at distances much less than the de Sitter radius, and ii) allow local and covariant techniques for defining the Wick powers described below eq. (2.22). These demands drive us to consider the class of states which are “globally Hadamard” [43].⁴

⁴ A simple example of non-Hadamard states in dS QFT are the Mottola-Allen or α -vacua [49, 41] (excepting the Euclidean state). These states satisfy S1 and S2, but the correlation functions of these states possess off lightcone singularities and differ in the character of their lightcone singularity from that of the standard Minkowski vacuum at arbitrarily short distances [50]. The expectation value of the stress tensor in such states has infinite fluctuations [51]. Of course, the “states of interest” depend on the context, and interesting things have been done with graviton α -vacua [52].

For scalar field theory a quasi-free Hadamard state is one for which the 2-pt function has no spacelike singularities and has a prescribed singularity behavior at null separations. More precisely, the nature of a distribution's singularity may be described by its wave front set (WF) [53]. A scalar 2-pt function $\Delta(x_1, x_2)$ is Hadamard if its wave front set is given by [54]

$$\text{WF}(\Delta) = \{(x_1, k_1; x_2, -k_2) \in (T^*dS_D \setminus \{0\})^2 \mid (x_1, k_1) \sim (x_2, k_2), k_1 \in V_1^+\}, \quad (2.24)$$

where $(x_1, k_1) \sim (x_2, k_2)$ denotes that x_1 and x_2 may be joined by a null geodesic k_1 and k_2 are cotangent and coparallel to that geodesic. In this paper we will deal with tensor-valued fields. We will refer to a graviton state as Hadamard if, in addition to being quasi-free, the state's 2-pt function has no spacelike singularities and its wave front set is given by (2.24), where the wave front set of tensor distribution is simply the union of the the wave front sets of its components in a local trivialization [53]. This definition is sufficient to guarantee the desired properties i) and ii) above and is the natural generalization of the Hadamard condition for vector fields [55, 56]. The Hadamard condition for tensor distributions may also be formulated in terms of "Hadamard's fundamental solutions" [57, 58].

Gauge symmetry

Let us now address the gauge symmetry of linearized gravity (2.14). While in principal the algebraic approach allows us to quantize the theory without gauge fixing – see e.g. [59] for an example in the case of $U(1)$ gauge fields on curved spacetimes – in practice, however, it is more practical to partially gauge fix, i.e. to choose a gauge condition which admits some residual gauge redundancy but which is sufficient to allow us to invert the equations of motion. We will utilize two types of gauges. The first is the class of generalized de Donder gauges [36] which satisfy the gauge condition

$$\nabla^\nu h_{\mu\nu}(x) - \frac{\beta}{2} \nabla_\mu h(x) = 0, \quad \beta \in \mathbb{R}. \quad (2.25)$$

The choice $\beta = 1$ is typically referred to as de Donder or harmonic gauge while $\beta = 0$ is transverse gauge. At least for generic values of β we expect to be able to impose this gauge condition non-linearly. The de Donder gauges (2.25) do not completely fix the gauge freedom as it is possible to construct vector fields $\xi^\mu(x)$ such that $\nabla_{(\mu} \xi_{\nu)}(x)$ satisfy (2.25).

In linearized gravity one may use the linearized equations of motion to impose further gauge conditions; in particular, one may impose that solutions to the equations of motion be in transverse traceless (TT) gauge:

$$\nabla^\nu h_{\mu\nu}(x) = 0, \quad h(x) = 0. \quad (2.26)$$

On dS there exist vector fields $\xi^\mu(x)$ such that $\nabla_{(\mu} \xi_{\nu)}(x)$ satisfy (2.26) (see, e.g., [60] or Appendix F of [33]), so there still exists residual gauge symmetry in TT gauge.⁵ For most

⁵The gauge may be fixed completely by imposing, e.g., transverse traceless synchronous gauge where in addition to (2.26) one imposes $h_{t\mu}(x) = 0$ for some time coordinate t , but this introduces a preferred timelike direction which is undesirable for our investigation.

of our analysis below we adopt TT gauge as we work exclusively at in the linearized theory and this gauge makes our analysis rather simple. In §5 we compute the 2-pt function of the state Ω in the de Donder gauges as well so that it may be utilized in future perturbative treatments of non-linear gravity.

A very useful fact is that in any gauge, when acting on solutions to the linearized equations of motion, the set of test functions \mathcal{T}_T defined in (2.21) may be further restricted to the class of TT test functions [37]

$$\mathcal{T}_{TT} := \{f^{\mu\nu} \in C_0^\infty(T_0^2(dS_D)) \mid f^{\mu\nu} = f^{\nu\mu}, \nabla_\mu f^{\mu\nu} = 0, g_{\mu\nu} f^{\mu\nu} = 0\}. \quad (2.27)$$

3 The state Ω

In this section we construct the maximally-symmetric state Ω by computing the 2-pt function $\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega$ in TT gauge. Our derivation of $\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega$ is straight-forward but it utilizes a great amount of simple technology which we spend the next three subsections describing. We finally compute $\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega$ in §3.4, as well as verify the Hadamard and positivity properties of this 2-pt function. We finish this section by contrasting our result with earlier works in §3.5. Henceforth we set the de Sitter radius $\ell = 1$.

3.1 Transverse traceless projection operator

A convenient way to impose the transverse and traceless conditions on the metric perturbation is via a transverse traceless projection operator [35]. It is natural to construct this operator from the linearized Weyl tensor; from the symmetries of the Weyl tensor it follows that the operation $\nabla^\gamma \nabla^\delta C_{\gamma\mu\delta\nu}^{(1)}(h)$ constructs from any symmetric tensor $h_{\alpha\beta}(x)$ a symmetric, rank-2 TT tensor. Therefore we define the TT projection operator $P_{\mu\nu}^{\alpha\beta}$ via

$$P_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} := \nabla^\gamma \nabla^\delta C_{\gamma\mu\delta\nu}^{(1)}(h). \quad (3.1)$$

This operator is symmetric on each pair of indices $P_{\mu\nu}^{\alpha\beta} = P_{\mu\nu}^{(\alpha\beta)} = P_{(\mu\nu)}^{\alpha\beta}$ and is transverse and traceless on indices μ, ν :

$$\nabla^\nu P_{\mu\nu}^{\alpha\beta} f_{\alpha\beta} = 0, \quad g^{\mu\nu} P_{\mu\nu}^{\alpha\beta} f_{\alpha\beta} = 0. \quad (3.2)$$

Additional properties of $P_{\mu\nu}^{\alpha\beta}$ we will need are [35]:

- i) the d'Alembertian commutes with $P_{\mu\nu}^{\alpha\beta}$, i.e. $\square P_{\mu\nu}^{\alpha\beta} f_{\alpha\beta}(x) = P_{\mu\nu}^{\alpha\beta} \square f_{\alpha\beta}(x)$,
- ii) the action of $P_{\mu\nu}^{\alpha\beta}$ on TT tensors is

$$P_{\mu\nu}^{\alpha\beta} w_{\alpha\beta}(x) = -\frac{1}{2} \frac{(D-3)}{(D-2)} (\square - 2)(\square - D) w_{\mu\nu}(x), \quad (3.3)$$

iii) if the product $p^{\mu\nu}(x)f_{\alpha\beta}(x)$ is compactly supported then $P_{\mu\nu}^{\alpha\beta}$ is self-adjoint in the sense that

$$\int_x p^{\mu\nu}(x)P_{\mu\nu}^{\alpha\beta}f_{\alpha\beta}(x) = \int_x f^{\mu\nu}(x)P_{\mu\nu}^{\alpha\beta}p_{\alpha\beta}(x). \quad (3.4)$$

Obviously, the projection operator annihilates any tensor $q_{\mu\nu}(x)$ for which $\nabla^\mu\nabla^\sigma C_{\mu\nu\sigma\rho}^{(1)}(q) = 0$. This includes total derivative and metric terms

$$P_{\mu\nu}^{\alpha\beta}\nabla_\alpha f_\beta(x) = 0, \quad P_{\mu\nu}^{\alpha\beta}(f(x)g_{\alpha\beta}) = 0, \quad (3.5)$$

as for these terms $C_{\alpha\beta\gamma\delta}^{(1)}(q) = 0$ (recall (2.15) and (2.16)). In addition we determine from (3.3) that the projection operator annihilates TT solutions to the de Sitter Fierz-Pauli equation [61]

$$(\square - M^2 - 2)h_{\mu\nu}(x) = 0, \quad (3.6)$$

(the factor of 2 arising from the cosmological constant term in the action) with mass values of $M^2 = 0$ and $M^2 = (D - 2)$. Solutions for $M^2 = 0$ are solutions to linearized Einstein equations (2.12) and satisfy $\nabla^\alpha C_{\alpha\beta\gamma\delta}^{(1)}(q) = 0$; solutions for $M^2 = (D - 2)$ are sometimes called “partially massless,” correspond to the Higuchi lower bound for unitary massive spin-2 fields [62], and satisfy $\nabla^\alpha\nabla^\gamma C_{\alpha\beta\gamma\delta}^{(1)}(q) = 0$.

3.2 Maximally symmetric bi-tensors

It is convenient to work with maximally symmetric bi-tensors (MSBTs) as these manifestly preserve maximal symmetry (classic references on these objects include [63, 64, 65, 66]). The tensor structures of MSBTs at x, \bar{x} are constructed by taking covariant derivatives of $Z := Z(x, \bar{x})$ in the tangent spaces of x and \bar{x} respectively. For rank-2 symmetric MSBTs there are five allowed index structures:

$$t_{\mu\nu}^{(1)\bar{\mu}\bar{\nu}} := g_{\mu\nu}g^{\bar{\mu}\bar{\nu}}, \quad (3.7)$$

$$t_{\mu\nu}^{(2)\bar{\mu}\bar{\nu}} := (\nabla_{(\mu}\nabla^{(\bar{\mu}}Z)(\nabla_{\nu)}\nabla^{\bar{\nu}})Z, \quad (3.8)$$

$$t_{\mu\nu}^{(3)\bar{\mu}\bar{\nu}} := (\nabla_{(\mu}Z)(\nabla^{(\bar{\mu}}Z)(\nabla_{\nu)}\nabla^{\bar{\nu}})Z, \quad (3.9)$$

$$t_{\mu\nu}^{(4)\bar{\mu}\bar{\nu}} := (\nabla_\mu Z)(\nabla_\nu Z)(\nabla^{\bar{\mu}}Z)(\nabla^{\bar{\nu}}Z), \quad (3.10)$$

$$t_{\mu\nu}^{(5)\bar{\mu}\bar{\nu}} := [g_{\mu\nu}(\nabla^{\bar{\mu}}Z)(\nabla^{\bar{\nu}}Z) + (\nabla_\mu Z)(\nabla_\nu Z)g^{\bar{\mu}\bar{\nu}}]. \quad (3.11)$$

Any rank-2 symmetric MSBT may be written in terms of these tensors with five scalar coefficient functions of Z , i.e.,

$$M_{\mu\nu}^{\bar{\mu}\bar{\nu}}(Z) = \sum_{i=1}^5 a_i(Z)t_{\mu\nu}^{(i)\bar{\mu}\bar{\nu}}. \quad (3.12)$$

However, often this is not the most convenient way of organizing the five scalar functions which determine $M_{\mu\nu}^{\bar{\mu}\bar{\nu}}(Z)$; following [67], we note that the most general $M_{\mu\nu}^{\bar{\mu}\bar{\nu}}(Z)$ may

also be written in the forms

$$M_{\mu\nu}^{\bar{\mu}\bar{\nu}}(Z) = b_j(Z)t_{\mu\nu}^{(j)\bar{\mu}\bar{\nu}} + b_1(Z)t_{\mu\nu}^{(1)\bar{\mu}\bar{\nu}} + \sum_{i=1}^3 G_{\mu\nu}^{(i)\bar{\mu}\bar{\nu}}(c_i(Z)), \quad j = 2 \text{ or } 4, \quad (3.13)$$

where the $G_{\mu\nu}^{(i)\bar{\mu}\bar{\nu}}(c_i(Z))$ are MSBTs which are total derivatives at x and/or \bar{x} :

$$G_{\mu\nu}^{(1)\bar{\mu}\bar{\nu}}(c_1(Z)) := [g_{\mu\nu}\nabla^{\bar{\mu}}\nabla^{\bar{\nu}} + g^{\bar{\mu}\bar{\nu}}\nabla_{\mu}\nabla_{\nu}] c_1(Z), \quad (3.14)$$

$$G_{\mu\nu}^{(2)\bar{\mu}\bar{\nu}}(c_2(Z)) := \nabla_{(\mu} [c_2(Z)(\nabla^{\bar{\mu}}Z)(\nabla_{\nu)}\nabla^{\bar{\nu}}Z)] + (x \leftrightarrow \bar{x}), \quad (3.15)$$

$$G_{\mu\nu}^{(3)\bar{\mu}\bar{\nu}}(c_3(Z)) := \nabla_{(\mu} [c_3(Z)(\nabla_{\nu)}Z)(\nabla^{\bar{\mu}}Z)(\nabla^{\bar{\nu}}Z)] + (x \leftrightarrow \bar{x}). \quad (3.16)$$

By expanding the expressions (3.14)-(3.16) one may readily verify that $b_j(Z)$, $b_1(Z)$, and the three $c_i(Z)$ uniquely determine the five $a_i(Z)$ in (3.12).

Tensor indices belonging the same tangent space are easily contracted and may be simplified by noting that $\nabla_{\mu}X^A$, $A = 0, \dots, D$ are conformal Killing vectors on dS_D :

$$\nabla_{\mu}\nabla_{\nu}X^A = -X^A g_{\mu\nu}, \quad \Rightarrow \quad \nabla_{\mu}\nabla_{\nu}Z = -Z g_{\mu\nu}, \quad (3.17)$$

and as a result Z is a maximally symmetric bi-scalar eigenfunction of the d'Alembertian:

$$\square Z = \bar{\square} Z = -DZ. \quad (3.18)$$

Other useful contractions that follow from (3.18) include:

$$\begin{aligned} (\nabla^{\mu}Z)(\nabla_{\mu}Z) &= (1 - Z^2), \\ (\nabla^{\mu}Z)(\nabla_{\mu}\nabla_{\bar{\nu}}Z) &= -Z(\nabla_{\bar{\nu}}Z), \\ (\nabla^{\mu}Z)(\nabla^{\bar{\nu}}Z)(\nabla_{\mu}\nabla_{\bar{\nu}}Z) &= -Z(1 - Z^2). \end{aligned} \quad (3.19)$$

MSBTs may also be parametrized in terms of the parallel propagator $g_{\mu\bar{\nu}}(Z)$ and the unit normal vector $n_{\mu}(Z)$ which is tangent to the shortest geodesic between x and \bar{x} [63]. These objects are defined in terms of derivatives of Z as follows:

$$g_{\mu\bar{\nu}}(Z) := (1 + Z)\nabla_{\mu}\nabla_{\bar{\nu}}\ln(1 + Z) = \nabla_{\mu}\nabla_{\bar{\nu}}Z - \frac{1}{(1 + Z)}(\nabla_{\mu}Z)(\nabla_{\bar{\nu}}Z). \quad (3.20)$$

$$n_{\mu}(Z) := \frac{-1}{(1 - Z^2)^{1/2}}\nabla_{\mu}Z. \quad (3.21)$$

An advantage of these variables is that they are bounded functions of Z ; a disadvantage is that their covariant derivatives are cumbersome. Using the parallel propagator one may translate the tensor structure of a tensor in the tangent space of \bar{x} to that of x , but the resulting object is still a function of \bar{x} , e.g., $g_{\mu}^{\bar{\nu}}V_{\bar{\nu}}(\bar{x}) = V_{\mu}(\bar{x})$.

Often we will only need to know an MSBT modulo terms proportional to the metric or terms which are total derivatives at x or \bar{x} . We refer to such terms as ‘metric’ and ‘grad’ terms respectively. For instance, we may write the most general MSBT (3.13) as

$$M_{\mu\nu}^{\bar{\mu}\bar{\nu}}(Z) = b_2(Z)(\nabla_{(\mu}\nabla^{\bar{\mu}}Z)(\nabla_{\nu)}\nabla^{\bar{\nu}}Z) + \text{metric} + \text{grad}. \quad (3.22)$$

Later it will be useful to note that

$$(\nabla_{(\mu} \nabla^{(\bar{\mu}} Z)(\nabla_{\nu)} \nabla^{\bar{\nu}} Z) = \text{metric} + \text{grad}, \quad Z(\nabla_{(\mu} \nabla^{(\bar{\mu}} Z)(\nabla_{\nu)} \nabla^{\bar{\nu}} Z) = \text{metric} + \text{grad}, \quad (3.23)$$

and that these are the only tensors with index structure $(\nabla_{(\mu} \nabla^{(\bar{\mu}} Z)(\nabla_{\nu)} \nabla^{\bar{\nu}} Z)$ which are “pure metric + grad.”

3.3 Källen-Lehmann representations

We will write the graviton 2-pt function in terms of relatively simple scalar functions which are closely related to the Green’s functions of the scalar Klein-Gordon equation. It is convenient to employ a kind of Källen-Lehmann representation for these scalar functions (utilized previously by [68, 16, 69, 70]). We review this Källen-Lehmann representation now.

The maximally-symmetric Green’s functions to the Klein-Gordon equation were obtained in ancient times by Motolla [49] and Allen [41]. Consider the function

$$\Delta_{\sigma}(Z) := \frac{1}{(4\pi)^{D/2}} \frac{\Gamma(-\sigma)\Gamma(\sigma + D - 1)}{\Gamma(\frac{D}{2})} {}_2F_1 \left[-\sigma, \sigma + D - 1; \frac{D}{2}; \frac{1 + Z}{2} \right], \quad (3.24)$$

where ${}_2F_1[a, b; c; z]$ is the Gauss hypergeometric function [71]. Eq. (3.24) defines an analytic function of Z in the complex Z plane cut along $Z \in [1, \infty)$; it is also a meromorphic function of σ with simple poles at $\sigma = 0, 1, 2, \dots$ and $\sigma = -(D - 1), -D, -(D + 1), \dots$. We obtain Green’s functions by adding various $i\epsilon$ prescriptions for avoiding the cut in the Z plane. Relating the parameter σ to the mass via

$$M^2 = M^2(\sigma) := -\sigma(\sigma + D - 1), \quad (3.25)$$

the distributions $\Delta_{\sigma}(Z + i\epsilon)$ and $\Delta_{\sigma}(Z + i\epsilon s)$ satisfy

$$(\square - M^2)\Delta_{\sigma}(Z + i\epsilon) = i \frac{\delta^D(x, \bar{x})}{\sqrt{-g}}, \quad (\square - M^2)\Delta_{\sigma}(Z + i\epsilon s) = 0, \quad (3.26)$$

where $s = s(x, \bar{x})$. The normalization of $\Delta_{\sigma}(Z)$ has been chosen so that these objects correspond to the time- and Wightman-ordered 2-pt functions of a canonically normalized massive scalar field on dS_D . $\Delta_{\sigma}(Z)$ is a scalar Hadamard distribution.

Perhaps the simplest example of a Källen-Lehmann representation is that for $\Delta_{\sigma}(Z)$ itself [68]:

$$\Delta_{\sigma}(Z) = \int_{C_{\sigma}} \frac{d\omega}{2\pi i} \frac{(2\omega + D - 1)}{(\omega - \sigma)(\omega + \sigma + D - 1)} \Delta_{\omega}(Z). \quad (3.27)$$

The integration contour C_{σ} is traversed from $-i\infty$ to $+i\infty$ within the strip $\text{Re } \sigma < \text{Re } \omega < 0$

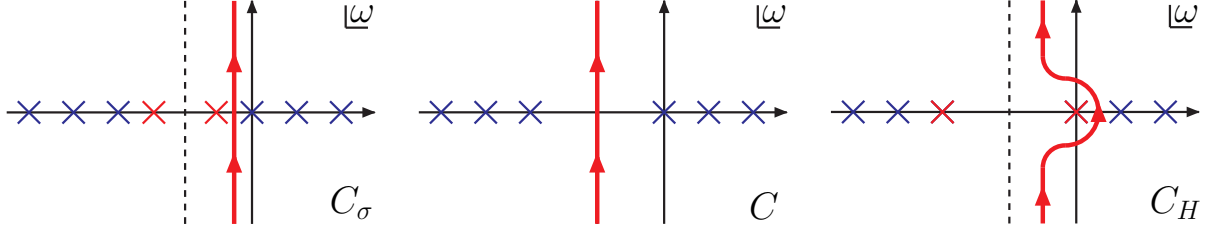


Figure 3: Integration contours in the complex ω plane. For each plot the dashed line denotes $\text{Re } \omega = -(D-1)/2$, blue crosses denote the locations of singularities in $\Delta_\omega(Z)$, and red crosses denote locations of other singularities of the integrand.

– see Fig. 3. By acting with $(\square - M^2)$ we obtain the identity

$$\begin{aligned}
i \frac{\delta^D(x, \bar{x})}{\sqrt{-g}} &= \int_{C_\sigma} \frac{d\omega}{2\pi i} \frac{(2\omega + D - 1)}{(\omega - \sigma)(\omega + \sigma + D - 1)} (\square - M^2(\sigma)) \Delta_\omega(Z + i\epsilon) \\
&= \int_{C_\sigma} \frac{d\omega}{2\pi i} \frac{(2\omega + D - 1)}{(\omega - \sigma)(\omega + \sigma + D - 1)} \left[-(\omega - \sigma)(\omega + \sigma + D - 1) \Delta_\omega(Z + i\epsilon) \right. \\
&\quad \left. + i \frac{\delta^D(x, \bar{x})}{\sqrt{-g}} \right] \\
&= - \int_C \frac{d\omega}{2\pi i} (2\omega + D - 1) \Delta_\omega(Z + i\epsilon).
\end{aligned} \tag{3.28}$$

In the second equality the second term on the right-hand side does not contribute because for this term the integration contour may be closed in the right half-plane without acquiring any residues. In the final equality C is a contour in the ω plane traversed from $-i\infty$ to $+i\infty$ which crosses the real line within the strip $-(D-1) < \text{Re } \omega < 0$ – see Fig. 3. By changing the cut prescription $i\epsilon \rightarrow i\epsilon s$ we deduce

$$- \int_C \frac{d\omega}{2\pi i} (2\omega + D - 1) \Delta_\omega(Z + i\epsilon s) = 0, \tag{3.29}$$

which vanishes as a distribution.

It is well-known that there does not exist a maximally-symmetric solution to the massless Klein-Gordon equation [72]. This is due to the factor $\Gamma(\sigma)$ in (3.24): in the limit $M^2 \rightarrow 0$

$$\Delta_\sigma(Z) = \frac{1}{M^2 \text{vol}(S^D)} + \mathcal{O}(M^0), \quad M^2 \ll 1, \tag{3.30}$$

where $\text{vol}(S^D) = 2\pi^{(D+1)/2}/\Gamma(\frac{D+1}{2})$ is the volume of a unit S^D . Thus $\Delta_\sigma(Z)$ is undefined for $M^2 = 0$. Consider instead the maximally-symmetric solution to the equation [73]

$$(\square - M^2) H_\sigma(Z + i\epsilon s) = \frac{1}{\text{vol}(S^D)}. \tag{3.31}$$

The solution to this equation is simply $H_\sigma(Z + i\epsilon s) = \Delta_\sigma(Z + i\epsilon s) - \frac{1}{M^2 \text{vol}(S^D)}$. Unlike $\Delta_\sigma(Z)$ the function $H_\sigma(Z)$ is regular in the neighborhood of $\sigma = 0$, so $H_0(Z)$ exists and satisfies

$$\square H_0(Z + i\epsilon s) = \frac{1}{\text{vol}(S^D)}. \quad (3.32)$$

Since $H_\sigma(Z)$ differs from $\Delta_\sigma(Z)$ only by an additive constant it is also a Hadamard distribution, and remains so in the $\sigma \rightarrow 0$ limit. $H_0(Z)$ may also be written as the limit

$$H_0(Z) = \left[\frac{\partial}{\partial(M^2)} (M^2 \Delta_\sigma(Z)) \right]_{M^2=0}, \quad (3.33)$$

or may be written as a contour integral

$$H_0(Z) = \int_{C_H} \frac{d\omega}{2\pi i} \frac{(2\omega + D - 1)}{\omega(\omega + D - 1)} \Delta_\omega(Z), \quad (3.34)$$

where C_H is a contour in the complex ω plane that is traversed from $-i\infty$ to $+i\infty$ mostly in the left half-plane, but is deformed so as to keep the pole at $\omega = 0$ to the left of the contour – see Fig. 3. To verify this expression, let us act with the d'Alembertian:

$$\begin{aligned} \square H_0(Z + i\epsilon s) &= \int_{C_H} \frac{d\omega}{2\pi i} \frac{(2\omega + D - 1)}{\omega(\omega + D - 1)} \square \Delta_\omega(Z + i\epsilon s) \\ &= - \int_{C_H} \frac{d\omega}{2\pi i} (2\omega + D - 1) \Delta_\omega(Z + i\epsilon s) \\ &= - \int_C \frac{d\omega}{2\pi i} (2\omega + D - 1) \Delta_\omega(Z + i\epsilon s) + \frac{1}{\text{vol}(S^D)} \\ &= + \frac{1}{\text{vol}(S^D)}. \end{aligned} \quad (3.35)$$

To obtain the third equality we deform the contour from C_H to C ; along the way we pick up a residue from the simple pole at $\omega = 0$ contained in $\Delta_\omega(Z)$. The final equality follows from (3.29). By changing the $i\epsilon$ prescription we obtain a distribution which satisfies $\square H_0(Z + i\epsilon) = \frac{1}{\text{vol}(S^D)} + i\delta^D(x, \bar{x})/\sqrt{-g}$.

3.4 Transverse traceless 2-pt function

We are finally ready to combine our ingredients to compute the 2-pt function $\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega$. In TT gauge this object satisfies the equation of motion

$$\int_x \int_{\bar{x}} f^{\mu\nu}(x) p_{\bar{\mu}\bar{\nu}}(\bar{x}) \frac{1}{2} (\square - 2) \langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega = 0, \quad f, p \in \mathcal{T}_{\text{TT}}. \quad (3.36)$$

The most general *manifestly* maximally-symmetric ansatz for $\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega$ may be written

$$\begin{aligned} \langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega &= \Delta_{\mu\nu}^{\text{TT} \bar{\mu}\bar{\nu}}(Z + i\epsilon s) \\ &:= P_{\mu\nu}^{\alpha\beta} P_{\bar{\alpha}\bar{\beta}}^{\bar{\mu}\bar{\nu}} \left[A(Z + i\epsilon s) (\nabla_{(\alpha} \nabla^{(\bar{\alpha}} Z) (\nabla_{\beta)} \nabla^{\bar{\beta}} Z) \right]. \end{aligned} \quad (3.37)$$

The term in brackets is consistent with the most general symmetric rank-2 MSBT (recall (3.22) and the fact that the projection operators annihilate grad and metric terms). This ansatz could be relaxed in several ways and still yield a maximally-symmetric state, but (3.37) is the simplest and will be sufficient for our purposes.

It is simple to obtain the equation of motion for $A(Z + i\epsilon s)$ from (3.36). First note that because the d'Alembertian commutes with the projection operators

$$\frac{1}{2}(\square - 2)\Delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(Z + i\epsilon s) = P_{\mu\nu}{}^{\alpha\beta}P_{\bar{\alpha}\bar{\beta}}^{\bar{\mu}\bar{\nu}}\frac{1}{2}(\square - 2)\left[A(Z + i\epsilon s)(\nabla_{(\alpha}\nabla^{\bar{\alpha}}Z)(\nabla_{\beta)}\nabla^{\bar{\beta}}Z)\right]. \quad (3.38)$$

Noting that for any bi-scalar $F(x, \bar{x})$

$$\square\left[F(x, \bar{x})(\nabla_{(\alpha}\nabla^{\bar{\alpha}}Z)(\nabla_{\beta)}\nabla^{\bar{\beta}}Z)\right] = [(\square + 2)F(x, \bar{x})](\nabla_{(\alpha}\nabla^{\bar{\alpha}}Z)(\nabla_{\beta)}\nabla^{\bar{\beta}}Z) + \text{grad} + \text{metric}, \quad (3.39)$$

we simplify (3.38) to

$$\frac{1}{2}(\square - 2)\Delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(Z + i\epsilon s) = P_{\mu\nu}{}^{\alpha\beta}P_{\bar{\alpha}\bar{\beta}}^{\bar{\mu}\bar{\nu}}\left[\frac{1}{2}\square A(Z + i\epsilon s)\right](\nabla_{(\alpha}\nabla^{\bar{\alpha}}Z)(\nabla_{\beta)}\nabla^{\bar{\beta}}Z). \quad (3.40)$$

Upon inserting this expression into (3.36), we use the self-adjointness of the projection operators (3.4), as well as their action on TT tensors (3.3), to obtain

$$0 = \int_x \int_{\bar{x}} f^{\mu\nu}(x)p_{\bar{\mu}\bar{\nu}}(\bar{x})\left\{\left[\frac{(D-3)^2}{8(D-2)^2}\square^2(\square - (D-2))\bar{\square}(\bar{\square} - (D-2))A(Z + i\epsilon s)\right](\nabla_{(\mu}\nabla^{\bar{\mu}}Z)(\nabla_{\nu)}\nabla^{\bar{\nu}}Z)\right\} \quad (3.41)$$

In order for the right-hand side of this equality to vanish the term in curly braces must be composed solely of grad and metric terms. The most general such tensor with the index structure of $(\nabla_{(\mu}\nabla^{\bar{\mu}}Z)(\nabla_{\nu)}\nabla^{\bar{\nu}}Z)$ is $(a + bZ)(\nabla_{(\mu}\nabla^{\bar{\mu}}Z)(\nabla_{\nu)}\nabla^{\bar{\nu}}Z)$ with a and b arbitrary constants. Therefore we obtain the equation of motion

$$\frac{(D-3)^2}{8(D-2)^2}\square^3(\square - (D-2))^2A(Z + i\epsilon s) = a + bZ, \quad (3.42)$$

for arbitrary a, b .

We now show that there exists a maximally-symmetric solution to (3.42) for $a = 1/\text{vol}(S^D)$ and for arbitrary b . For the moment consider the case $b = 0$; then the maximally-symmetric solution is

$$A(Z + i\epsilon s) = \frac{8(D-2)^2}{(D-3)^2} \int_{C_A} \frac{d\omega}{2\pi i} \frac{(2\omega + D - 1)}{\omega^3(\omega + D - 1)^3(\omega + 1)^2(\omega + D - 2)^2} \Delta_\omega(Z + i\epsilon s). \quad (3.43)$$

where the integration contour $C_A := C_H$ is the integration contour used in (3.34) – see also Fig. 4. The denominator of the integrand is simply the eigenvalue of $\Delta_\omega(Z + i\epsilon s)$ with respect

to the derivative operator on the left-hand side of (3.42). Let us explicitly verify that this is a solution to (3.42) with $a = 1/\text{vol}(S^D)$:

$$\begin{aligned}
& \frac{(D-3)^2}{8(D-2)^2} \square^3 (\square - (D-2))^2 A(Z + i\epsilon s) \\
&= \int_{C_A} \frac{d\omega}{2\pi i} \frac{(2\omega + D - 1)}{\omega^3 (\omega + D - 1)^3 (\omega + 1)^2 (\omega + D - 2)^2} \square^3 (\square - (D-2))^2 \Delta_\omega(Z + i\epsilon s) \\
&= - \int_{C_A} \frac{d\omega}{2\pi i} (2\omega + D - 1) \Delta_\omega(Z + i\epsilon s) \\
&= \text{Res} [(2\omega + D - 1) \Delta_\omega(Z + i\epsilon s)]_{\omega=0} - \int_C \frac{d\omega}{2\pi i} (2\omega + D - 1) \Delta_\omega(Z + i\epsilon s) \\
&= \frac{1}{\text{vol}(S^D)}. \tag{3.44}
\end{aligned}$$

To obtain the third equality we deform the integration contour from C_A to C acquiring the residue of the pole at $\omega = 0$ contained in $\Delta_\omega(Z)$ along the way. The final equality follows from (3.29). It is this residue that sets the value $a = 1/\text{vol}(S^D)$. We may also construct solutions to (3.42) with $a = 1/\text{vol}(S^D)$ and $b \neq 0$ by adding to (3.43) a solution to (3.42) with $a = 0$ and $b \neq 0$. Recalling that $\square Z = -DZ$ we see that the solution to this equation is proportional to Z . However, the projection operators which act on $A(Z + i\epsilon s)$ annihilate the term $Z(\nabla_{(\mu} \nabla^{(\bar{\mu}} Z)(\nabla_{\nu)} \nabla^{\bar{\nu}} Z)$ so this additional term does not contribute to $\Delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(Z + i\epsilon s)$. Therefore the parameter b represents a redundancy of description introduced by our use of the projection operators. Similarly, we could add to $A(Z + i\epsilon s)$ a constant as the projection operators annihilate the term $(\nabla_{(\mu} \nabla^{(\bar{\mu}} Z)(\nabla_{\nu)} \nabla^{\bar{\nu}} Z)$. For simplicity we keep (3.43) as our expression for $A(Z + i\epsilon s)$.⁶

The graviton 2-pt function simplifies considerably when smeared against test functions and using this expression it is easy to verify that $\Delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(Z + i\epsilon s)$ is positive and Hadamard. For $f, p \in \mathcal{T}_{\text{TT}}$:

$$\begin{aligned}
\langle h(f)h(p) \rangle_\Omega &= \int_x \int_{\bar{x}} f^{\mu\nu}(x) p_{\bar{\mu}\bar{\nu}}(\bar{x}) \Delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(Z + i\epsilon s) \\
&= 2 \int_x \int_{\bar{x}} f^{\mu\nu}(x) p_{\bar{\mu}\bar{\nu}}(\bar{x}) H_0(Z + i\epsilon s) (\nabla_{(\mu} \nabla^{(\bar{\mu}} Z)(\nabla_{\nu)} \nabla^{\bar{\nu}} Z). \tag{3.46}
\end{aligned}$$

Here we have again utilized (3.4) and (3.3) and we have also recognized $H_0(Z)$ (3.34). By

⁶If we wish, we may also express $A(Z)$ as a sum of terms of the form

$$A(Z) = \sum_{i=0}^4 c_i \left[\left(\frac{\partial}{\partial \sigma} \right)^i (M^2(\sigma) \Delta_\sigma(Z)) \right]_{\sigma=0} + \sum_{j=0}^2 k_j \left[\left(\frac{\partial}{\partial \sigma} \right)^j D_\sigma(Z) \right]_{\sigma=-1}. \tag{3.45}$$

To do so we deform the integration contour from C_A to C , e.g. to $\text{Re } \omega = -(D-1)/2$, along the way acquiring residues from the poles at $\omega = 0$ and $\omega = -1$. These residues provide the terms (3.45) and the remaining contour integral, which is absolutely convergent, vanishes as the integrand is odd. We will not need the coefficients in (3.45) so we do not bother to record them.

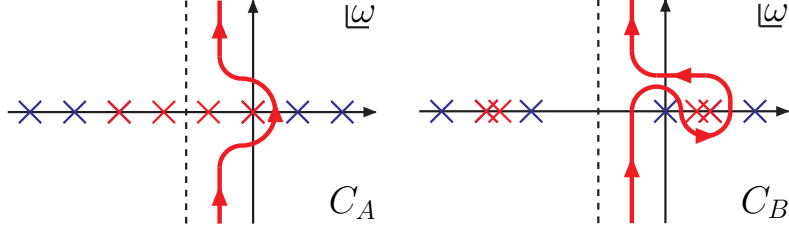


Figure 4: Integration contours in complex ω plane. For each plot the dashed line denotes $\text{Re } \omega = -(D-1)/2$, blue crosses denote the locations of singularities in $\Delta_\omega(Z)$, and red crosses denote locations of other singularities of the integrand. **Left:** the contour $C_A = C_H$ utilized for $A(Z)$ and $U(Z)$. **Right:** the contour utilized for $B(Z)$ in the de Donder gauge term (see §5). The location of the poles at $\omega = \sigma_\beta, -(\sigma_\beta + D-1)$ depicted correspond to transverse gauge $\beta = 0$.

noting that

$$\begin{aligned} (\nabla_{(\mu} \nabla^{(\bar{\mu}} Z)(\nabla_{\nu)} \nabla^{\bar{\nu}} Z) &= \frac{1}{4} \nabla_\mu \nabla_\nu \nabla^{\bar{\mu}} \nabla^{\bar{\nu}} Z^2 + \text{metric} \\ &= \frac{1}{4} \eta_{AC} \eta_{BD} (\nabla_\mu \nabla_\nu X^A X^B) (\nabla^{\bar{\mu}} \nabla^{\bar{\nu}} \bar{X}^A \bar{X}^B) + \text{metric}, \end{aligned} \quad (3.47)$$

where $\eta_{AB} = \text{diag}\{-, +, \dots, +\}$ is the metric of the Minkowski embedding space, we may use the chain rule to recast (3.46) as

$$\langle h(f) h(p) \rangle_\Omega = \frac{1}{2} \eta_{AC} \eta_{BD} \int_x \int_{\bar{x}} F^{AB}(x) P^{CD}(\bar{x}) H_0(Z + i\epsilon s) \quad (3.48)$$

with smearing functions

$$F^{AB}(x) := \nabla_\mu \nabla_\nu (f^{\mu\nu}(x) X^A X^B), \quad P^{CD}(\bar{x}) := \nabla^{\bar{\mu}} \nabla^{\bar{\nu}} (p_{\bar{\mu}\bar{\nu}}(\bar{x}) \bar{X}^C \bar{X}^D). \quad (3.49)$$

These test functions belong to the class

$$\mathcal{T}_S := \left\{ f(x) \in C_0^\infty(dS_D) \mid \int_x f(x) = 0 \right\}. \quad (3.50)$$

Bros et. al [73] have shown that $H_0(Z)$ is a positive kernel for this class of scalar test functions, so it follows that $\Delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(Z + i\epsilon s)$ is a positive kernel for the test functions \mathcal{T}_{TT} . That $\Delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(Z + i\epsilon s)$ is Hadamard follows from the fact that $H_0(Z + i\epsilon s)$ is Hadamard.

The normalization of our 2-pt function has been fixed by examining the time-ordered correlation function $\langle Th_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega$ which is obtained by changing the cut prescription: $\langle Th_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega = \Delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(Z + i\epsilon)$. This object satisfies

$$\int_x \int_{\bar{x}} f^{\mu\nu}(x) p_{\bar{\mu}\bar{\nu}}(\bar{x}) \frac{1}{2} (\square - 2) \langle Th_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega = i \int_x f^{\mu\nu}(x) p_{\mu\nu}(x), \quad f, p \in \mathcal{T}_{\text{TT}}, \quad (3.51)$$

or equivalently, the distribution equation

$$\frac{1}{2}(\square - 2)\langle Th_{\mu\nu}(x)h^{\mu\nu}(\bar{x})\rangle_{\Omega} = i\delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x}), \quad (3.52)$$

where $\delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x})$ is the identity operator on the space of test functions \mathcal{T}_{TT} . The identity operator satisfies

$$\int_x \int_{\bar{x}} f^{\mu\nu}(x) p_{\bar{\mu}\bar{\nu}}(\bar{x}) \delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x}) = \int_x f^{\mu\nu}(x) p_{\mu\nu}(x), \quad f, p \in \mathcal{T}_{\text{TT}}, \quad (3.53)$$

as well as

$$f_{\mu\nu}(x) = \int_{\bar{x}} \delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x}) q_{\bar{\mu}\bar{\nu}}(\bar{x}) \in \mathcal{T}_{\text{TT}}, \quad q \in C_0^\infty(T_2^0(dS_D)). \quad (3.54)$$

As defined by its action on test function $\delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x})$ is maximally-symmetric, though it need not be written manifestly so. E.g.,

Proposition 3.1 *The TT identity operator $\delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x})$ which satisfies (3.53) and (3.54) may be written*

$$\delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x}) = P_{\mu\nu}{}^{\alpha\beta} P_{\bar{\alpha}\bar{\beta}}^{\bar{\mu}\bar{\nu}} \left[U(x, \bar{x}) (\nabla_{(\alpha} \nabla^{\bar{\alpha}} Z) (\nabla_{\beta)} \nabla^{\bar{\beta}} Z) \right], \quad (3.55)$$

where $U(x, \bar{x})$ is any solution to the equation

$$\frac{(D-3)^2}{4(D-2)^2} \square(\square - (D-2)) \bar{\square}(\bar{\square} - (D-2)) U(x, \bar{x}) = a + bZ + i \frac{\delta^D(x, \bar{x})}{\sqrt{-g}}, \quad (3.56)$$

with arbitrary coefficients a and b .

The maximally-symmetric solution for $U(x, \bar{x})$ which satisfies (3.56) with $a = 1/\text{vol}(S^D)$, $b = 0$, is given by

$$U(Z + i\epsilon) = -\frac{4(D-2)^2}{(D-3)^2} \int_{C_A} \frac{d\omega}{2\pi i \omega^2} \frac{(2\omega + D - 1)}{(\omega + D - 1)^2 (\omega + 1)^2 (\omega + D - 2)^2} \Delta_\omega(Z + i\epsilon). \quad (3.57)$$

We obtain the normalization for the 2-pt function from this expression.

3.5 Comparison with previous works

We now discuss two points of contact between our result for the graviton 2-pt function and existing results in the literature. First, we note that our expression for the TT part of the 2-pt function agrees with result obtained by first constructing the 2-pt function on the Euclidean sphere S^D , then analytically continuing the result to Lorentz-signature. We show this in Appendix A. This procedure has been used previously by many authors [32, 13, 26, 25, 33]. The validity of this procedure for massless fields has been debated [34, 24].⁷ Since we have

⁷For interacting massive scalar QFTs there is no debate: this procedure has been shown to agree with Lorentz-signature constructions to all orders in perturbation theory [17, 18]. The massiveness of the fields is essential to the proof.

explicitly verified that our state is a positive and Hadamard, it would seem that the concerns of [34, 24] are not realized. One could have guessed this would be the case from the fact that the Euclidean action for gravitons in TT gauge is bounded below, and therefore the de Sitter version of the Osterwalder-Schrader theorem (see e.g. [74]) would seem to assure that the Lorentzian state defined by the analytically continued Euclidean 2-pt function is free of pathologies (e.g., the Lorentzian state is positive).

Now we discuss the claim of [35] that there does not exist a maximally-symmetric solution to the TT part of the graviton 2-pt Schwinger-Dyson equations. As stated this is clearly not true; the 2-pt function constructed in the previous section provides a counterexample and defines a healthy state. Below we summarize the difference between the calculation of [35] and our own, then show that the state defined by the less-symmetric 2-pt function of [35] is equivalent to Ω .

Ref. [35] utilizes the same ansatz (3.37) but with $A(Z)$ replaced by a function $\mathcal{S}_2(x, \bar{x})$ which is not assumed to be maximally-symmetric.⁸ These authors obtain the same equation of motion for $\mathcal{S}_2(x, \bar{x})$ (3.42); however, they only consider the values $a = 0$, $b = 0$, and as a result do not obtain the maximally-symmetric solution that exists for $a = 1/\text{vol}(S^D)$. The solution of [35] is explored in detail in [75] whose notation we now follow. The form of the solution is $\mathcal{S}_2(x, \bar{x}) = S_2(Z) + \delta S_2(x, \bar{x})$, where $S_2(Z) = A(Z)$ and $\delta S_2(x, \bar{x})$ is a symmetry-breaking term which is a solution to (3.42) with $a = -1/\text{vol}(S^D)$, $b = 0$. The authors choose $\delta S_2(x, \bar{x})$ so as to be invariant under spatial translations and rotations in the Poincaré chart; the explicit form of $\delta S_2(x, \bar{x})$ is given by eq. (84) of [75]. Ref. [75] also showed that $\delta S_2(x, \bar{x})$ is not annihilated by the projection operators in the ansatz (3.37).

Let us denote the state defined by the 2-pt function of [35] by MTW (after the authors). To see that MTW is equivalent to Ω examine the smeared 2-pt function $\langle h(f)h(p) \rangle_{\text{MTW}}$. Inserting $\mathcal{S}_2(x, \bar{x})$ into this expression and utilizing the self-adjointness of the projection operators (3.4) as well as their action on TT tensors (3.3) we obtain

$$\langle h(f)h(p) \rangle_{\text{MTW}} = \int_x \int_{\bar{x}} f^{\mu\nu}(x) p_{\bar{\mu}\bar{\nu}}(\bar{x}) [H_0(Z + i\epsilon s) + Q(x, \bar{x})] (\nabla_{(\mu} \nabla^{(\bar{\mu}} Z) (\nabla_{\nu)} \nabla^{\bar{\nu}} Z), \quad (3.58)$$

where

$$\begin{aligned} Q(x, \bar{x}) &:= \frac{(D-3)^2}{8(D-2)^2} \square(\square - (D-2)) \bar{\square}(\bar{\square} - (D-2)) \delta S_2(x, \bar{x}) \\ &= -\frac{1}{(D-1)\text{vol}(S^D)} [\ln \eta + \ln \bar{\eta}]. \end{aligned} \quad (3.59)$$

Here η is the time coordinate in the Poincaré chart of de Sitter (2.6). This function satisfies

$$\square Q(x, \bar{x}) = \bar{\square} Q(x, \bar{x}) = -\frac{1}{\text{vol}(S^D)}, \quad (3.60)$$

and as a result the term in brackets in (3.58) is a Hadamard bi-solution to the massless Klein-Gordon equation. By judicious use of the chain rule we show that $Q(x, \bar{x})$ does not

⁸Ref. [35] uses the distance measure $y(x, \bar{x}) = 2(1 - Z(x, \bar{x}))$, but for clarity we will continue to use $Z(x, \bar{x})$.

contribute to (3.58):

$$\begin{aligned}
\langle h(f)h(p) \rangle_{\text{MTW}} - \langle h(f)h(p) \rangle_{\Omega} &= \int_x \int_{\bar{x}} f^{\mu\nu}(x) p_{\bar{\mu}\bar{\nu}}(\bar{x}) Q(x, \bar{x}) (\nabla_{(\mu} \nabla^{\bar{\mu}} Z) (\nabla_{\nu)} \nabla^{\bar{\nu}} Z) \\
&= \int_x \int_{\bar{x}} f^{\mu\nu}(x) p_{\bar{\mu}\bar{\nu}}(\bar{x}) [\nabla_{(\mu} \nabla^{\bar{\mu}} Q(x, \bar{x})] Z (\nabla_{\nu)} \nabla^{\bar{\nu}} Z) \\
&= 0,
\end{aligned} \tag{3.61}$$

where the final equality follows from the fact that $\nabla_{\mu} \nabla^{\bar{\mu}} Q(x, \bar{x}) = 0$. This computation shows that the states MTW and Ω are equivalent as probed by *all* local observables $\mathcal{A}(dS_D)$ and thus are equivalent states within the algebraic framework.⁹ This computation also shows that the state MTW admits an analytic continuation from the algebra of observables of the Poincaré chart to the algebra of observables of the global chart, i.e. the entire de Sitter manifold.¹⁰

4 Consequences

The previous section examined the maximally-symmetric state Ω in great detail; in this section we use our results to establish a few basic properties of *generic* states. Recall that we may connect the notion of an algebraic state with the more familiar notion of a vector in a Hilbert space via the “GNS construction”: given an algebraic state Ω the GNS construction provides a Hilbert space \mathcal{H}_{Ω} containing a cyclic vector $|\Omega\rangle$, a representation of the abstract elements of the observable algebra (which we will not distinguish in notation), as well as a dense set of state vectors $\mathcal{D} \subset \mathcal{H}_{\Omega}$ constructed by application of the observable algebra on $|\Omega\rangle$, i.e.

$$\mathcal{D} := \{A |\Omega\rangle \mid A \in \mathcal{A}(dS_D)\}. \tag{4.1}$$

For technical reasons in this section we restrict attention to observables whose support is contained in a *contractible* region. We call a compact subset $\mathcal{O} \subset dS_D$ contractible if its boundary $\partial\mathcal{O}$ may be contracted to a point. We denote the resulting algebra by $\mathcal{A}_c(dS_D)$ and the dense set of states generated by $\mathcal{A}(dS_D)$ on \mathcal{H}_{Ω} by \mathcal{D}_c :

$$\mathcal{D}_c := \{A |\Omega\rangle \mid A \in \mathcal{A}_c(dS_D)\}. \tag{4.2}$$

Note that all compact co-dimension 1 surfaces in the Poincaré or static charts are contractible, so for quantization on these backgrounds $\mathcal{A}_c(dS_D) = \mathcal{A}(dS_D)$ and $\mathcal{D}_c = \mathcal{D}$; it is

⁹Ref. [76] has shown the lesser condition that $\delta S_2(x, \bar{x})$ does not contribute to the linearized Weyl tensor 2-pt function.

¹⁰ A related computation one could perform is to check that $\langle Th(f)h(p) \rangle_{\text{MTW}} - \langle Th(f)h(p) \rangle_{\Omega}$ is a homogeneous solution to the equations of motion. Indeed

$$-\frac{1}{2}(\square - 2) [\langle Th(f)h(p) \rangle_{\text{MTW}} - \langle Th(f)h(p) \rangle_{\Omega}] = 0 \tag{3.62}$$

holds as a distribution equation; that is, the left hand side vanishes as tested by any functions in \mathcal{T}_{TT} .

only on the global chart that $\mathcal{A}_c(dS_D) \subset \mathcal{A}(dS_D)$, $\mathcal{D}_c \subset \mathcal{D}$. We should emphasize that the set of states \mathcal{D}_c can approximate *any* state on \mathcal{H}_Ω and are not limited to highly-symmetric or “static” configurations.

We will use the following lemma of Higuchi frequently [37]:

Lemma 4.1 *Every $f \in \mathcal{T}_{\text{TT}}$ whose support is contained in a contractible region may be written*

$$f^\sigma{}_\nu(x) = \Upsilon_{[\gamma\delta][\mu\nu]}^{[\alpha\beta][\rho\sigma]} (\nabla_\rho \nabla^\mu + \delta^\mu_\rho) v_{\alpha\beta}{}^{\gamma\delta}(x), \quad (4.3)$$

where $\Upsilon_{[\gamma\delta][\mu\nu]}^{[\alpha\beta][\rho\sigma]}$ is as in (2.18) and $v \in C_0^\infty(T_2^2(dS_D))$ is a test tensor with the (anti-)symmetries and tracelessness of the Weyl tensor. The support of v may be made arbitrarily close to that of f .¹¹

As pointed out by Higuchi [37], it is an immediate consequence of this lemma that for every metric observable $h(f) \in \mathcal{A}_c(dS_D)$ there exists an equivalent linearized Weyl tensor observable $C^{(1)}(v) \in \mathcal{A}_c(dS_D)$ with f and v related as in (4.3).

4.1 The Reeh-Schlieder theorem

In this section we prove a version of the Reeh-Schlieder theorem (theorem 5.3.1 of [44]) for linearized gravity on de Sitter backgrounds:

Theorem 4.2 *The set of states*

$$\mathcal{A}_c(\mathcal{O})\Omega := \{A|\Omega\} \mid A \in \mathcal{A}_c(\mathcal{O})\} \quad (4.4)$$

generated from operators in a contractible subset $\mathcal{O} \subset dS_D$ is dense on \mathcal{D}_c .

The Reeh-Schlieder theorem has many important consequences – see e.g. [44, 77]. For instance, it follows from the theorem that any two states may be distinguished by comparing their correlation functions in any contractible subset \mathcal{O} – no two states may “look alike” as probed by *all* observables in $\mathcal{A}_c(\mathcal{O})$.¹² Our immediate purpose for introducing the Reeh-Schlieder theorem is that it allows us to describe any state in \mathcal{D}_c by a state in $\mathcal{A}_c(\mathcal{O})\Omega$, a feature we will need in §4.2. The Reeh-Schlieder theorem has been proven, under various assumptions, for both free and interacting QFTs on classes of curved backgrounds – see e.g. [78, 79, 80, 81, 82, 83, 84].

The Reeh-Schlieder theorem is a statement about the analyticity of n-pt functions of state vectors in \mathcal{D}_c . For scalar QFTs on Minkowski space the Reeh-Schlieder theorem may be proven by showing that n-pt functions are holomorphic on a sufficiently large domain in complex Minkowski space such that determining their value on an open region of the real Lorentzian section determines the n-pt functions everywhere on the section. There is the

¹¹Higuchi proves this statement for tensors whose support is contained in the Poincaré chart of de Sitter, but one may argue by continuity that the result holds for any tensor whose support is contractible.

¹²Of course, realistic observers with finite resources and precision do not have the access to all observables of the theory.

subtlety that the domain of holomorphicity does not actually contain the real Lorentzian section – there n-pt functions are distributions, i.e. “boundary values of holomorphic functions” – but this is accommodated for by the “edge-of-the-wedge” theorem [77]. For scalar QFTs on de Sitter there exists a rather straight-forward extension of this proof by Bros et. al. [85] which uses the holomorphicity properties of scalar n-pt functions on complexified de Sitter space (or equivalently, the complexified embedding space). In our expressions for the graviton 2-pt function obtained in §3.4 the functions $A(Z)$ and $H_0(Z)$ have the same domain of holomorphicity as the 2-pt functions of Klein-Gordon fields on dS_D ,¹³ so it appears that it would be rather simple to extend the proof of [85] to linearized gravity. Ultimately we chose another path for our proof, but we mention this tactic because it provides a nice geometric description of the underlying physics.

A faster if less picturesque approach to proving the Reeh-Schlieder theorem is to employ the tools of microlocal analysis. In particular, the analyticity properties of a distribution may be characterized by its analytic wave front set (WF_A).¹⁴ In this language, one expects the Reeh-Schlieder theorem to hold on a dense set of states whose correlation functions have analytic wave front sets that are “sufficiently small,” indicating a high degree of analyticity. There exists a powerful microlocal version of the edge-of-the-wedge theorem, namely Proposition 5.3 of [82]:

Proposition 4.3 *Let M be a real analytic connected manifold and $u \in D'(M)$ a distribution with the property that*

$$\text{WF}_A(u) \cap -\text{WF}_A(u) = \emptyset. \quad (4.5)$$

Then for each non-void open subset $\mathcal{O} \subset M$ if the restriction of u to \mathcal{O} vanishes then $u = 0$.

Using this proposition [82] proved the Reeh-Schlieder theorem for scalar QFTs on real analytic spacetimes whose state n-pt functions satisfy (4.5). Below we show that the analytic wave front sets of linearized Weyl tensor n-pt functions of states in \mathcal{D}_c also satisfy (4.5). Following [82] we may then quickly prove the Reeh-Schlieder theorem for linearized gravitons on dS_D .

Lemma 4.4 *Consider two state vectors $|\Psi\rangle, |\Theta\rangle \in \mathcal{D}_c$. The analytic wave front set of the amplitudes*

$$G_{\Psi\Theta}^m := \langle \Psi | C^{(1)\alpha_1}_{\beta_1\gamma_1\delta_1}(x_1) \dots C^{(1)\alpha_n}_{\beta_n\gamma_n\delta_n}(x_n) | \Theta \rangle \quad (4.6)$$

satisfy (4.5).

Proof. We begin by determining the analytic wave front set of the graviton n-pt functions of Ω :

$$F_{\Omega\Omega}^n := \langle h_{\mu_1\nu_1}(x_1) \dots h_{\mu_n\nu_n}(x_n) \rangle_{\Omega}. \quad (4.7)$$

¹³In particular, they satisfy the “weak spectral condition” of [85]. This fact is not surprising given that both $A(Z)$ and $H_0(Z)$ are closely related to the Klein-Gordon 2-pt function $\Delta_{\sigma}(Z)$.

¹⁴A mathematical introduction to this technology may be found in Ch. 9 of [53]. For introductions to the applications to QFT see [48, 86].

Since Ω is quasi-free $F_{\Omega\Omega}^n$ vanishes for n odd while for n even

$$F_{\Omega\Omega}^n = \sum_P \prod_{r \in P} \langle h_{\mu_{r1}\nu_{r1}}(r_1) h_{\mu_{r2}\nu_{r2}}(r_2) \rangle_{\Omega}. \quad (4.8)$$

Here P denotes the a partition of the set $\{x_1, \dots, x_n\}$ into pairs of points labelled $r = (r_1, r_2)$; the points are labelled so as to preserve the Wightman operator ordering. The analytic wave front set of the 2-pt function is easily determined from that of $H_0(Z)$:

$$\begin{aligned} \Xi(x_1, k_1; x_2, k_2) &:= \text{WF}_A(F_{\Omega\Omega}^2) \\ &= \{(x_1, k_1; x_2, -k_2) \in (T^*dS_D \setminus \{0\})^2 \mid (x_1, k_1) \sim (x_2, k_2), k_1 \in V_1^+\}. \end{aligned} \quad (4.9)$$

Here we have once again used the notation that $(x_1, k_1) \sim (x_2, k_2)$ denotes that x_1 and x_2 may be joined by a null geodesic k_1 and k_2 are cotangent and coparallel to that geodesic. Then the analytic wave front set of $F_{\Omega\Omega}^n$ is

$$\text{WF}_A(F_{\Omega\Omega}^n) = \cup_P \oplus_{r \in P} \Xi(x_{r1}, k_{r1}; x_{r2}, k_{r2}). \quad (4.10)$$

Next we consider the linearized Weyl tensor n-pt functions $G_{\Omega\Omega}^n$ which may be constructed from the $F_{\Omega\Omega}^n$ by repeated use of (2.17). Using the basic facts that i) $\text{WF}_A(\partial u) \subseteq \text{WF}_A(u)$, and ii) $\text{WF}_A(fu) \subseteq \text{WF}_A(u)$ for $f \in C^\infty$ we readily determine that

$$\text{WF}_A(G_{\Omega\Omega}^n) \subseteq \text{WF}_A(F_{\Omega\Omega}^n) = \cup_P \oplus_{r \in P} \Xi(x_{r1}, k_{r1}; x_{r2}, k_{r2}). \quad (4.11)$$

From this we determine that $\text{WF}_A(G_{\Omega\Omega}^n)$ satisfy (4.5). The amplitude $G_{\Psi\Theta}^n$ may be constructed from a finite number of $F_{\Omega\Omega}^m$ with $m \geq n$ by smearing operators with contractible C_0^∞ test functions; it follows that if all $\text{WF}_A(F_{\Omega\Omega}^m)$ satisfy (4.5) so too do all $\text{WF}(G_{\Psi\Theta}^n)$. ■

Proof of Theorem 4.2. Consider the amplitudes $\langle \Psi | A | \Omega \rangle$ with $A \in \mathcal{A}_c(\mathcal{O})$ and \mathcal{O} a contractible region on dS_D . Due to Lemma 4.1 we may restrict attention to the linearized Weyl tensor correlators

$$\langle \Psi | C^{(1)}(v_1) \dots C^{(1)}(v_n) | \Omega \rangle, \quad \text{supp } v_i \subseteq \mathcal{O}. \quad (4.12)$$

Let $|\Psi\rangle$ be in the orthogonal complement to $\mathcal{A}_c(\mathcal{O})\Omega$ so that (4.12) vanishes for all v_i . It follows that the restriction of each $G_{\Psi\Omega}^n$ to \mathcal{O} vanishes. Due to Lemma 4.4 we may use Proposition 4.3 to conclude that each $G_{\Psi\Omega}^n = 0$ on all of $(dS_D)^n$. Then all correlation functions of Ψ vanish and hence $\Psi = 0$, i.e. the orthogonal complement of $\mathcal{A}_c(\mathcal{O})\Omega$ is empty. ■

4.2 Cosmic no-hair theorem

We are almost ready to state the no-hair theorem. The remaining ingredient we need is the cluster decomposition of Ω correlation functions of local observables. Cluster decomposition does *not* hold for the unsmeared graviton correlation function $\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_{\Omega}$. Recall that we may write

$$\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_{\Omega} = H_0(Z) (\nabla_{(\mu} \nabla^{(\bar{\mu}} Z) (\nabla_{\nu)} \nabla^{\bar{\nu}} Z) + \text{grad} + \text{metric}. \quad (4.13)$$

At large $|Z| \gg 1$ the function $H_0(Z)$ has the asymptotic expansion

$$H_0(Z) = c_1 \ln(1 + Z) + c_2 + \mathcal{O}(Z^{-2}), \quad |Z| \gg 1, \quad (4.14)$$

where c_1, c_2 are coefficients whose value is unimportant now. The c_2 term is pure grad + metric (recall (3.23)). By judicious use of the chain rule we obtain

$$\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega = [-c_1 \ln(1 + Z) + \mathcal{O}(Z^{-2})] g_{(\mu}^{\bar{\mu}} g_{\nu)}^{\bar{\nu})} + \text{grad} + \text{metric}, \quad (4.15)$$

where $g_{\mu\bar{\nu}}$ is the parallel propagator (3.20) which is $\mathcal{O}(Z^0)$ at large $|Z| \gg 1$. Therefore for $|Z| \gg 1$ the 2-pt function behaves like $\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_\Omega \sim \ln Z$ as expected for a massless spin-2 field.

However, the Ω correlation functions of local observables $A \in \mathcal{A}_c(dS_D)$ do enjoy a de Sitter version of cluster decomposition associated with large timelike and achronal separations [37]. Consider the 2-pt function $\langle h(f)h(p) \rangle_\Omega$ for two observables that are well-separated in the sense that $|Z_{\min}| \gg 1$ where

$$Z_{\min} = \min\{Z(x, \bar{x}) \mid x \in \text{supp } f^{\mu\nu}(x), \bar{x} \in \text{supp } p^{\bar{\mu}\bar{\nu}}(\bar{x})\}. \quad (4.16)$$

Large $|Z| \gg 1$ corresponds to large timelike or achronal separation. From Lemma 4.1 this 2-pt function may be recast as linearized Weyl tensor correlation function $\langle C^{(1)}(v)C^{(1)}(u) \rangle_\Omega$ with appropriate v, u . We may then use the known asymptotic behavior of the linearized Weyl tensor 2-pt function to bound this 2-pt function [87, 88, 76]:¹⁵

Proposition 4.5 *The linearized Weyl tensor 2-pt function of the state Ω may be written*

$$\left\langle C^{(1)\alpha}_{\beta\gamma\delta}(x) C^{(1)\bar{\alpha}}_{\bar{\beta}\bar{\gamma}\bar{\delta}}(\bar{x}) \right\rangle_\Omega = \sum_i F_i(Z) T^{(i)\alpha}_{\beta\gamma\delta}{}^{\bar{\alpha}}_{\bar{\beta}\bar{\gamma}\bar{\delta}}(Z), \quad (4.17)$$

with $F_i(Z)$ that satisfy

$$F_i(Z) < \frac{c}{|Z|^2}, \quad \text{for } |Z| \gg 1, \quad (4.18)$$

with c a finite constant and tensors $T^{(i)}(Z)$ composed of the parallel propagator and normal vectors defined in (3.20) and (3.21).

The parallel propagator and normal vectors are $\mathcal{O}(Z^0)$ for $|Z| \gg 1$ and so it follows that we may bound

$$\langle h(f)h(p) \rangle_\Omega < \frac{c}{|Z_{\min}|^2}, \quad |Z_{\min}| \gg 1, \quad h(f), h(p) \in \mathcal{A}_c(dS_D), \quad (4.19)$$

where c is a finite constant that depends on the test functions f, p but not on Z_{\min} .

The cosmic no-hair theorem follows immediately from (4.19):

¹⁵ The most explicit expression available for the linearized Weyl 2-pt function in the form (4.17) are given in the Corrigendum of [87] for $D = 4$ dimensions. For other spacetime dimensions one may obtain (4.17) from eq. (94) of [88].

Theorem 4.6 *Let $\Psi \in \mathcal{A}_c(\mathcal{O})\Omega$, \mathcal{O} a contractible subset of dS_D , and $A(f) \in \mathcal{A}_c(dS_D)$ with f schematically denoting the test function. Then*

$$|\langle A(f) \rangle_\Psi - \langle A(f) \rangle_\Omega| < \frac{c}{|Z_{\min}|^2}, \quad \text{for } |Z_{\min}| \gg 1, \quad (4.20)$$

where

$$Z_{\min} = \min\{Z(x, \bar{x}) \mid x \in \text{supp } f(x), \bar{x} \in \mathcal{O}\}. \quad (4.21)$$

Here c is a finite constant that depends on the test function f and state Ψ but does not depend on Z_{\min} .

Proof. Let $|\Psi\rangle = B|\Omega\rangle$, $B \in \mathcal{A}_c(\mathcal{O})$. Like all states $|\Psi\rangle$ is normalized, so $\langle\Omega| B^* B |\Omega\rangle = 1$. The correlation function $\langle A(f) \rangle_\Psi = \langle\Omega| B^* A(f) B |\Omega\rangle$ is constructed out of smeared products of the graviton 2-pt function. From (4.19) it follows that any such 2-pt function connecting $A(f)$ and B is $\mathcal{O}(|Z_{\min}|^2)$. Thus

$$\begin{aligned} \langle A(f) \rangle_\Psi &= \langle\Omega| B^* A(f) B |\Omega\rangle \\ &= \langle\Omega| B^* B |\Omega\rangle \langle\Omega| A(f) |\Omega\rangle + \mathcal{O}(|Z_{\min}|^{-2}) \\ &= \langle A(f) \rangle_\Omega + \mathcal{O}(|Z_{\min}|^{-2}), \end{aligned} \quad (4.22)$$

which verifies (4.20). ■

The implications of this theorem are plain to see. Consider an arbitrary state $|\Psi\rangle \in \mathcal{D}_c$. Using the Reeh-Schlieder property we may write this state as $|\Psi\rangle = B|\Omega\rangle$, $B \in \mathcal{A}_c(\mathcal{O})$ with \mathcal{O} a contractible region centered around the point $(\eta_i, \vec{0})$ in the Poincaré chart (2.6). Here η_i is a finite but arbitrarily early time. Now consider any observable $A \in \mathcal{A}_c(dS_D)$ whose support is centered around $(\eta_f, \vec{0})$ with $\eta_f \gg \eta_i$. For sufficiently large $\eta_f - \eta_i$ the embedding distance between all points in the support of A and all points in \mathcal{O} is large and Theorem 4.6 states that

$$|\langle A(\tau) \rangle_\Psi - \langle A(\tau) \rangle_\Omega| < c e^{-2\tau}, \quad (4.23)$$

where τ is the proper time separation between $(\eta_i, \vec{0})$ and $(\eta_f, \vec{0})$. We have used the fact that $|Z| \sim e^\tau$ at large timelike separations. Obviously (4.23) remains true if A is displaced from the origin of the spatial slice so long as it remains well within the causal future of \mathcal{O} .

5 de Donder 2-pt functions

In this section we change gears and construct the graviton 2-pt function of the state Ω in the class of generalized de Donder (dD) gauges defined by the gauge condition (2.25). Our purpose is to ease the tension in the literature regarding the existence of manifestly maximally-symmetric 2-pt functions for this useful class of gauges. The graviton 2-pt function in these gauges was examined earlier in [35, 75] for $\beta = 1$ and in [36] for all β . These works contend that there does not exist a maximally-symmetric 2-pt function for gauge parameter $\beta < 2$, including transverse gauge $\beta = 0$. On the other hand, for $D = 4$ the

generalized dD gauge 2-pt functions may be inferred from the covariant gauge graviton 2-pt function of [25, 26, 27]; from this procedure one finds a maximally-symmetric dD 2-pt functions for all but a discrete set of β (this set does not include transverse gauge).¹⁶ Furthermore, it is rather straight-forward to use Euclidean techniques like those presented in Appendix A to construct the 2-pt function in transverse gauge. Indeed, below we obtain maximally-symmetric 2-pt functions for all but a discrete set of β for all $D \geq 4$, consistent with [25, 26, 27]. We have not checked that all aspects of our result for $D = 4$ matches that of [25]. We use Lorentz-signature techniques and follow essentially the same steps as [35, 36] until the point at which the analysis of these works goes awry.

In order to construct an ansatz for the 2-pt function let us first note that any TT tensor trivially satisfies the dD gauge condition (2.25) for all β . Therefore our ansatz for the dD 2-pt function contains the TT part obtained in §3.4 as well as a second term:

$$\langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_{\Omega} = \Delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(Z + i\epsilon s) + \Delta_{\mu\nu}^{\text{dD}\bar{\mu}\bar{\nu}}(Z + i\epsilon s). \quad (5.1)$$

From the general form of an MSBT (3.13) it follows that the dD term may of the form

$$\Delta_{\mu\nu}^{\text{dD}\bar{\mu}\bar{\nu}}(Z) = C(Z) g_{\mu\nu} g^{\bar{\mu}\bar{\nu}} + \text{grad}, \quad (5.2)$$

where the grad part is determined from $C(Z)$ by imposing the gauge condition (2.25). This is easily accomplished by using the dD projection operator [36]

$$P_{\mu\nu}^{(\beta)} \phi(x) := (\nabla_{\mu} \nabla_{\nu} + a_{\beta} g_{\mu\nu} + b_{\beta} g_{\mu\nu} \square) \phi(x) = (\nabla_{\mu} \nabla_{\nu} + b_{\beta} g_{\mu\nu} (\square - M_{\beta}^2)) \phi(x), \quad (5.3)$$

with

$$a_{\beta} = \frac{2(1-D)}{2-\beta D} = (1 + D b_{\beta}), \quad b_{\beta} = \frac{\beta-2}{2-\beta D}, \quad M_{\beta}^2 = -\frac{a_{\beta}}{b_{\beta}} = \frac{2(D-1)}{\beta-2}. \quad (5.4)$$

For any scalar field $\phi(x)$ the tensor $P_{\mu\nu}^{(\beta)} \phi(x)$ satisfies (2.25). Therefore our ansatz for $\Delta_{\mu\nu}^{\text{dD}\bar{\mu}\bar{\nu}}(Z)$ is

$$\Delta_{\mu\nu}^{\text{dD}\bar{\mu}\bar{\nu}}(Z) := P_{\mu\nu} P^{\bar{\mu}\bar{\nu}} B(Z). \quad (5.5)$$

A rather awkward feature of the dD gauges is that in general the linearized equations of motion do not preserve the gauge condition. Recall that $L_{\mu\nu}^{(1)}(q)$ is transverse for any $q_{\mu\nu}(x)$, so the action of $L_{\mu\nu}^{(1)}$ preserves the gauge condition only for transverse gauge $\beta = 0$. Noting that

$$L_{\mu\nu}^{(1)}(\text{grad}) = 0, \quad L_{\mu\nu}^{(1)}(g\phi) = \frac{D-2}{2} P_{\mu\nu}^{(0)} \phi(x), \quad (5.6)$$

where

$$P_{\mu\nu}^{(0)} \phi(x) = (\nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} (\square + D - 1)) \phi(x) \quad (5.7)$$

constructs a transverse tensor from a scalar, we obtain

$$L_{\mu\nu}^{(1)}(P^{(\beta)} \phi) = \frac{D-2}{2} b_{\beta} P_{\mu\nu}^{(0)} (\square - M_{\beta}^2) \phi(x). \quad (5.8)$$

¹⁶For each set of authors the obstruction to obtaining maximally-symmetric 2-pt functions vanishes in the flat-space limit $\ell \rightarrow \infty$, so neither result contradicts known Minkowski results.

So, in admittedly awkward notation, the action of the linearized equations of motion on our ansatz is

$$L_{\mu\nu}^{(1)}(\Delta^{\text{dD}\overline{\mu\nu}}) = \frac{D-2}{2} b_\beta P_{\mu\nu}^{(0)} P^{(\beta)\overline{\mu\nu}} [(\square - M_\beta^2) B(Z)]. \quad (5.9)$$

We now proceed to obtain the equation of motion for $B(Z + i\epsilon s)$. The graviton 2-pt function satisfies the equation of motion

$$\int_x \int_{\bar{x}} f^{\mu\nu}(x) p_{\overline{\mu\nu}}(\bar{x}) \langle L_{\mu\nu}^{(1)}(h) h^{\overline{\mu\nu}}(\bar{x}) \rangle_\Omega = 0, \quad p \in \mathcal{T}_T, \quad f \in C_0^\infty(T_0^2(dS_D)). \quad (5.10)$$

The most general such $f^{\mu\nu}(x)$ may be uniquely decomposed into the parts [89]

$$f_{\mu\nu}(x) = f_{\mu\nu}^{\text{TT}}(x) + \nabla_{(\mu} \xi_{\nu)}(x) + g_{\mu\nu} f_1(x) + \nabla_\mu \nabla_\nu f_2(x), \quad (5.11)$$

where $f_{\mu\nu}^{\text{TT}}(x)$ is transverse traceless and $\xi_\nu(x)$ is transverse. Due to the self-adjointness of $L_{\mu\nu}^{(1)}$ neither the $\xi^\mu(x)$ nor $f_2(x)$ term contribute to (5.10). Similarly, using (5.9), the self-adjointness of $P_{\mu\nu}^{(0)}$, and the fact that $P^{(0)\mu\nu} f_{\mu\nu}^{\text{TT}}(x) = 0$ it follows that $f_{\mu\nu}^{\text{TT}}(x)$ does not contribute to (5.10). Therefore (5.10) is non-trivial only for test functions of the form $f^{\mu\nu}(x) = f_1(x) g^{\mu\nu}$. Noting that

$$g^{\mu\nu} P_{\mu\nu}^{(0)} \phi(x) = (1-D)(\square + D)\phi(x), \quad (5.12)$$

we obtain the equation

$$\frac{(1-D)(D-2)}{2} b_\beta \int_{\bar{x}} p_{\overline{\mu\nu}}(\bar{x}) P^{(\beta)\overline{\mu\nu}} [(\square + D)(\square - M_\beta^2) B(Z + i\epsilon s)] = 0. \quad (5.13)$$

It is important to note that the $P_{\mu\nu}^{(\beta)} Z = 0$ so the term in brackets is defined only up to a term linear in Z . Finally, noting that

$$P_{\overline{\mu\nu}}^{(\beta)} = b_\beta g_{\overline{\mu\nu}} (\overline{\square} - M_\beta^2) + \text{grad}, \quad (5.14)$$

(5.13) implies the following equation of motion for $B(Z + i\epsilon s)$:

$$\frac{(1-D)(D-2)}{2} \frac{(\beta-2)^2}{(2-\beta D)^2} (\square + D) (\square - M_\beta^2) (\overline{\square} - M_\beta^2) B(Z + i\epsilon s) = cZ, \quad (5.15)$$

where c is an arbitrary constant. We have inserted the value of b_β from (5.3). To this point our analysis agrees with that of [36], except that these authors only considered the case of $c = 0$.

There exists a maximally-symmetric solution to (5.15) with $c = -(D+1)/\text{vol}(S^D)$. We may construct this solution using the same Källen-Lehmann technique used in §3.4: defining σ_β via $M_\beta^2 = -\sigma_\beta(\sigma_\beta + D - 1)$, the solution is simply

$$B(Z) = \frac{2}{(1-D)(D-2)} \frac{(2-\beta D)^2}{(\beta-2)^2} \int_{C_B} \frac{d\omega}{2\pi i} \frac{(2\omega + D - 1)}{(\omega - 1)(\omega + D)(\omega - \sigma_\beta)^2(\omega + \sigma_\beta + D - 1)^2} \Delta_\omega(Z), \quad (5.16)$$

with integration contour C_B depicted in Fig. 4, i.e., it is traversed from $-i\infty$ to $+i\infty$ mostly in the left half-plane but is deformed so as to keep the poles at $\omega = 0, 2, 3, 4, \dots$ to the right and the poles at $\omega = 1, -D, \sigma_\beta, -(\sigma_\beta + D - 1)$ to the left. We may explicitly verify that (5.16) solves (5.15):

$$\begin{aligned}
& \frac{(1-D)(D-2)}{2} \frac{(\beta-2)^2}{(2-\beta D)^2} (\square + D) (\square - M_\beta^2)^2 B(Z + i\epsilon s) \\
&= \int_{C_B} \frac{d\omega}{2\pi i} \frac{(2\omega + D - 1)}{(\omega - 1)(\omega + D)(\omega - \sigma_\beta)^2(\omega + \sigma_\beta + D - 1)^2} (\square + D) (\square - M_\beta^2)^2 \Delta_\omega(Z + i\epsilon s) \\
&= - \int_{C_B} \frac{d\omega}{2\pi i} (2\omega + D - 1) \Delta_\omega(Z + i\epsilon s) \\
&= \text{Res} [(2\omega + D - 1) \Delta_\omega(Z + i\epsilon s)]_{\omega=1} - \int_C \frac{d\omega}{2\pi i} (2\omega + D - 1) \Delta_\omega(Z + i\epsilon s) \\
&= - \frac{(D+1)}{\text{vol}(S^D)} Z.
\end{aligned} \tag{5.17}$$

To obtain the third equality we deform the contour from C_B to C (see Fig. 3) acquiring the residue of the simple pole at $\omega = 1$. The final equality follows from simplifying the residue (which includes letting $\epsilon \rightarrow 0$ since this term has no cut in the Z plane) and recalling that the second term on the third line vanishes as a distribution. The contour C_B exists so long as the left and right poles do not overlap, i.e. so long as

$$M_\beta^2 = \frac{2(D-1)}{\beta-2} = -\sigma_\beta(\sigma_\beta + D - 1) \neq -n(n + D - 1), \quad \text{for } n = 0, 2, 3, 4, \dots \tag{5.18}$$

To summarize, excepting the values of β listed in (5.18), the graviton 2-pt function for the state Ω in dD gauge is manifestly maximally-symmetric and is given by (5.1) with $\Delta_{\mu\nu}^{\text{dD}\overline{\mu\nu}}(Z + i\epsilon s)$ as in (5.5) and $B(Z + i\epsilon s)$ as in (5.16). Just as in TT gauge, the time-ordered 2-pt function $\langle Th_{\mu\nu}(x) h^{\overline{\mu\nu}}(\bar{x}) \rangle_\Omega$ is provided from the Wightman-ordered 2-pt function simply by changing the $i\epsilon$ prescription $i\epsilon s \rightarrow i\epsilon$.

It is important to note that $\Delta_{\mu\nu}^{\text{dD}\overline{\mu\nu}}(Z + i\epsilon s)$ is “pure gauge” and as a result the addition of $\Delta_{\mu\nu}^{\text{dD}\overline{\mu\nu}}(Z + i\epsilon s)$ to the 2-pt function does not alter the positive or Hadamard attributes of Ω . Recall that because $\Delta_{\mu\nu}^{\text{dD}\overline{\mu\nu}}(Z + i\epsilon s)$ solves the linearized equations of motion we may use these equations to restrict the class of test functions used to construct $h(f)$ correlators to \mathcal{T}_{TT} ; however, $\Delta_{\mu\nu}^{\text{dD}\overline{\mu\nu}}(Z + i\epsilon s)$ vanishes against any $f \in \mathcal{T}_{\text{TT}}$ because it is composed solely of gradient and metric terms. Likewise $\Delta_{\mu\nu}^{\text{dD}\overline{\mu\nu}}(Z + i\epsilon s)$ does not contribute to correlators of the linearized Weyl tensor.

We conclude by contrasting our results with those of [35] and in particular [36] which considered dD gauge for general β . The analysis of [36] differs from the ours in two respects. First, this work only considers the equation of motion (5.15) for $B(Z + i\epsilon s)$ with $c = 0$. We have shown that the maximally-symmetric solution to (5.15) corresponds to $c = -(D+1)/\text{vol}(S^D)$. For this reason [36] was forced to consider solutions to (5.15) which have less symmetry. It turns out that for $\beta > 2$ the piece of their solution which is not maximally-symmetric is annihilated by the projection operators $P_{\mu\nu}^{(\beta)}$ and does not contribute to what we call $\Delta_{\mu\nu}^{\text{dD}\overline{\mu\nu}}(Z + i\epsilon s)$.

For $\beta \leq 2$ a second issue arises. Ref. [36] solves (5.15) by combining three scalar Green's function of the Klein-Gordon equation, one with mass $M^2 = -D$ and two with $M^2 = M_\beta^2 = 2(D-1)/(\beta-2)$. The former mass is always tachyonic while the latter is tachyonic for $\beta < 2$. The maximally-symmetric Green's functions to the Klein-Gordon equation $\Delta_\sigma(Z)$ (3.24) exist for all masses $M^2 \neq -n(n+D-1)$, $n \in \mathbb{N}_0$, and so could have been used for this purpose. However, the authors chose instead to use a non-maximally-symmetric Green's function for all tachyonic masses. As a result they obtain solutions with symmetry-breaking terms, and in this case these terms are not annihilated by the projection operators and remain in $\Delta_{\mu\nu}^{\text{dD}\bar{\mu}\bar{\nu}}(Z + i\epsilon s)$.

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A The Euclidean TT 2-pt function

In this appendix we compute the graviton 2-pt function in TT gauge on the Euclidean sphere S^D using standard harmonic analysis. We then show that the analytic continuation of this 2-pt function to Lorentz signature agrees with the 2-pt functions obtained in §3.4. Our Euclidean analysis is very similar to that of [65, 13, 25, 26, 33]. We set the radius of S^D to unity.

We denote scalar harmonics on S^D by $Y_{\vec{L}}(x)$; these harmonics are eigenfunctions of the Laplace operator

$$\square Y_{\vec{L}}(x) = -L(L+D-1)Y_{\vec{L}}(x), \quad (\text{A.1})$$

and are labelled by their angular momenta $\vec{L} = (L := L_D, L_{D-1}, \dots, L_1)$ which satisfy $L_D \geq L_{D-1} \geq \dots \geq L_2 \geq |L_1| \geq 0$. They form an orthonormal and complete set with respect to the scalar L^2 inner product:

$$\int_x Y_{\vec{L}}(x) Y_{\vec{M}}^*(x) = \delta_{\vec{L}\vec{M}}, \quad \sum_{\vec{L}} Y_{\vec{L}}(x) Y_{\vec{L}}^*(\bar{x}) = \frac{\delta^D(x, \bar{x})}{\sqrt{g}}. \quad (\text{A.2})$$

In this Appendix $\int_x F(x) = \int d^D x \sqrt{\gamma(x)} F(x)$ where $\gamma_{\mu\nu}$ is the metric on S^D . Similarly, we denote symmetric, transverse, traceless rank-2 tensor harmonics by $T_{\mu\nu}^{(\vec{L};\alpha)}(x)$; these are also eigenfunctions of the Laplace operator

$$\square T_{\mu\nu}^{(\vec{L};\alpha)}(x) = [-L(L+D-1) + 2] T_{\mu\nu}^{(\vec{L};\alpha)}(x), \quad (\text{A.3})$$

and are labelled by their angular momenta $\vec{L} = (L := L_D, L_{D-1}, \dots, L_1)$ which satisfy $L_D \geq L_{D-1} \geq \dots \geq L_2 \geq |L_1| \geq 2$, as well as a polarization index $\alpha = 1, \dots, (D-1)(D-2)/2$.

They form an orthonormal and complete set for TT tensors with respect to the tensor L^2 inner product:

$$\int_x T_{\mu\nu}^{(\vec{L};\alpha)}(x) (T^{(\vec{M};\beta)\mu\nu}(x))^* = \delta^{\vec{L}\vec{M}} \delta^{\alpha\beta}, \quad \sum_{\vec{L}} \sum_{\alpha} T_{\mu\nu}^{(\vec{L};\alpha)}(x) (T^{(\vec{L};\alpha)\bar{\mu}\bar{\nu}}(\bar{x}))^* =: \delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x}). \quad (\text{A.4})$$

The second expression defines $\delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x})$ on S^D . Explicit forms of the harmonics conforming to our conventions may be found in, e.g. [90, 91].

It is also useful to define the following maximally-symmetric bi-tensor harmonics. For scalars these are defined by

$$W_L(Z) := \sum_{\vec{j}} Y_{\vec{L}}(x) Y_{\vec{L}}^*(\bar{x}), \quad \vec{L} = (L, \vec{j}), \quad (\text{A.5})$$

and turn out to be just a polynomial in the Euclidean embedding distance $Z := Z(x, \bar{x}) = \cos \theta(x, \bar{x})$ where $\theta(x, \bar{x})$ is the angular separation between points on S^D . Explicitly [92],

$$W_L(Z) = \frac{(2L + D - 1) \Gamma(\frac{D-1}{2}) \Gamma(L + D - 1)}{4\pi^{(D+1)/2} \Gamma(D-1) \Gamma(L+1)} {}_2F_1 \left[-L, L + D - 1; \frac{D}{2}; \frac{1-Z}{2} \right]. \quad (\text{A.6})$$

Since L is an integer the hypergeometric function in this expression reduces to a polynomial of order L . Likewise for symmetric TT tensors we define

$$W_{\mu\nu}^L{}^{\bar{\mu}\bar{\nu}}(Z) := \sum_{\vec{j}} \sum_{\alpha} T_{\mu\nu}^{(L, \vec{j}, \alpha)}(x) (T_{(L, \vec{j}, \alpha)}^{\bar{\mu}\bar{\nu}}(\bar{x}))^*, \quad \vec{L} = (L, \vec{j}). \quad (\text{A.7})$$

A closed form for $W_{\mu\nu}^L{}^{\bar{\mu}\bar{\nu}}(Z)$ may be computed using the method of [66]; the result is

$$W_{\mu\nu}^L{}^{\bar{\mu}\bar{\nu}}(Z) = P_{\mu\nu}{}^{\alpha\beta} P_{\bar{\alpha}\bar{\beta}}^{\bar{\mu}\bar{\nu}} \left[N_L W_L(Z) (\nabla_{(\alpha} \nabla^{(\bar{\alpha}} Z) (\nabla_{\beta)} \nabla^{\bar{\beta})} Z) \right], \quad (\text{A.8})$$

with

$$N_L = \frac{4(D-3)^2}{(D-2)^2} \frac{1}{L^2(L+D-1)^2(L+1)^2(L+D)^2}. \quad (\text{A.9})$$

Clearly $W_L(Z)$ and $W_{\mu\nu}^L{}^{\bar{\mu}\bar{\nu}}(Z)$ are bi-eigenfunctions of the Laplacian with eigenvalues as in (A.1) and (A.3) respectively.

In TT gauge the graviton 2-pt function on S^D satisfies

$$\frac{1}{2}(\square - 2) \langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_{\text{E}} = -\delta_{\mu\nu}^{\text{TT}\bar{\mu}\bar{\nu}}(x, \bar{x}). \quad (\text{A.10})$$

It is easy to invert $-\frac{1}{2}(\square - 2)$ on the TT identity operator and simplify:

$$\begin{aligned} \langle h_{\mu\nu}(x) h^{\bar{\mu}\bar{\nu}}(\bar{x}) \rangle_{\text{E}} &= 2 \sum_{\vec{L}} \sum_{\alpha} \frac{T_{\mu\nu}^{(\vec{L}, \alpha)}(x) T_{(\vec{L}, \alpha)}^{*\bar{\mu}\bar{\nu}}(\bar{x})}{L(L+D-1)} \\ &= 2 \sum_{L=2}^{\infty} \frac{W_{\mu\nu}^L{}^{\bar{\mu}\bar{\nu}}(Z)}{L(L+D-1)} \\ &= P_{\mu\nu}{}^{\alpha\beta} P_{\bar{\alpha}\bar{\beta}}^{\bar{\mu}\bar{\nu}} \left[F(Z) (\nabla_{(\alpha} \nabla^{(\bar{\alpha}} Z) (\nabla_{\beta)} \nabla^{\bar{\beta})} Z) \right], \end{aligned} \quad (\text{A.11})$$

with

$$F(Z) = \frac{8(D-3)^2}{(D-2)^2} \sum_{L=2}^{\infty} \frac{W_L(Z)}{L^3(L+D-1)^3(L+1)^2(L+D)^2}. \quad (\text{A.12})$$

The sum of polynomials of Z in (A.12) converges for non-coincident points on S^D , i.e. for $Z \in [-1, 1)$; however, it does not converge for generic Z values outside this range. Since the analytic continuation of (A.11) amounts to extending the range of Z from $Z \in [-1, 1)$ to $Z \in \mathbb{C} \setminus [1, +\infty)$ we must first render $F(Z)$ into a more suitable form before the continuation. This may be done by a standard technique known as a Watson-Sommerfeld transformation [68] whereby the sum is recast as a contour integral. Consider the expression

$$F(Z) = -\frac{8(D-3)^2}{(D-2)^2} \int_{C_F} \frac{d\omega}{2\pi i} \frac{\pi}{\sin(\pi\omega)} \frac{W_\omega(-Z)}{\omega^3(\omega+D-1)^3(\omega+1)^2(\omega+D)^2}, \quad (\text{A.13})$$

where the integration contour C_F encloses in a clockwise fashion the poles in the integrand at $\omega = 2, 3, 4, \dots$ due to the factor of $\sin(\pi\omega)$ in the denominator. Using Cauchy's formula the contour integral is equivalent to the sum of residues due to these poles, and this sum is precisely (A.12). On the other hand, we may rewrite (A.13) by noting first that

$$\Delta_\omega(Z) = -\frac{\pi}{(2\omega+D-1)\sin\pi\omega} W_\omega(-Z), \quad (\text{A.14})$$

and second that the integrand decays sufficiently rapidly as $|\omega| \rightarrow \infty$ such that the contour may be deformed from C_F to C_A – see Fig. 4. After these manipulations the contour integral for $F(Z)$ is precisely that for $A(Z)$ in (3.43). The analytic continuation process is completed by adding the appropriate $i\epsilon$ prescription for avoiding the cut in $F(Z) = A(Z)$ along $Z \in [1, +\infty)$. Thus the analytic continuation of the graviton 2-pt function in TT gauge on S^D agrees with our result constructed explicitly on dS_D in §3.4.

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